Chapter 2

THE THEORY OF STEADY-STATE REACTIVE POWER CONTROL IN ELECTRIC TRANSMISSION SYSTEMS

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CONVENTIONS AND SYMBOLS

Reactive Power

In accordance with the widely used convention,

1. Reactive power at a generating station is
   positive if generated
   negative if absorbed.
2. Reactive power at a load is
   negative if generated
   positive if absorbed.
3. The receiving end of a transmission line is always treated as a load.

Per-Unit and Ordinary Units

Most equations are given in terms of positive-sequence phase-neutral quantities. The per-unit system is based on rated voltage V_0 and the surge impedance Z_0. Base power is therefore $3V_0^2/Z_0$ watts if V_0 is phase-neutral voltage; or V_0^2/Z_0 if V_0 is phase-phase voltage. In both cases base power is three-phase power.

Lower-case letters are sometimes used for per-unit voltages, currents, reactances, power, and so on. Inductances and capacitances in lower case are usually per-mile values. They may be in per-unit or in ordinary units.

Symmetrical and Radial Lines

The term *radial line* is used to describe a single transmission line with synchronous machines controlling the voltage at the sending end and no voltage control at the receiving end. The *symmetrical* line has identical synchronous machines at both ends, constraining the terminal voltages to be equal.

Emf and Voltage

The symbol E is used for an emf or *controlled* voltage.
The symbol V is used for voltage more generally.

Phasors are denoted by boldface type, for example, **E**

Primes indicate compensated or *virtual* quantities, for example, P_0'

Frequency is assumed to be 60 Hz.

Symbols

a	Line length, mi
B	Shunt susceptance, S
c	Capacitance per mile, F/mi (positive-sequence equivalent, phase-neutral)
E	(controlled) voltage
I	current, A
k_m	Mid-point compensation factor
k_{se}	Degree of series compensation
k_{sh}	Degree of shunt compensation
l	Inductance per mile, H/mi (positive-sequence equivalent, phase-neutral)
P	Power, W
P_0	Natural load or *Surge-Impedance load* (*SIL*), W
P_{max}	Maximum transmissible power, W
Q	Reactive power, VAr
s	Series compensation factor
V	Voltage

X	Reactance, Ω
Y	Admittance, S
Z	Impedance, Ω
Z_0	Characteristic or Surge Impedance, Ω
β	Wavenumber, radian/mi
δ	Transmission Angle, radian or degree
θ	Electrical length of line, radian
Γ	Propagation constant

Subscripts

r	Receiving end
s	Sending end
c	Capacitive
l	Inductive
γ	Compensator
0	Natural or characteristic

2.1. INTRODUCTION

2.1.1. Historical Background

The economics of ac power transmission have always forced the planning engineer to transmit as much power as possible through a given transmission line. Today, however, additional constraints loom much larger than they did in the past. First, the dependence of load centers on the continuity of electrical supplies has become more critical (as witnessed by events during the North-East power blackout of 1965 and the New York City blackout of 1977[6]). This means that the security, or reliability, of transmission circuits has needed to be continuously improved. Modern compensation methods have helped to make these improvements possible. Second, there has been extensive development of remote hydroelectric resources, such as the El Chocon-Cerros Colorados complex in Argentina, 1000 km from Buenos Aires, and the James Bay scheme in Québec, 1000 km from Montréal and Québec City.[16] Both these ac schemes are characterized not only by the long distances, but also by the large amounts of power to be transmitted (over 11,000 MW in the case of the James Bay scheme). The development of compensation schemes has helped to make ac transmission technically and economically competitive even in an age when the dc transmission alternative has made great strides also.

A third planning constraint has been the difficulty of acquiring right-of-way for new transmission circuits (the so-called corridor crisis). Increased pressure to maximize the utilization of both new and existing lines has helped to motivate the development and application of compensation systems.

This chapter begins with an account of the fundamental requirements in ac power transmission, *stability* and *voltage control.* The principles of operation of the main types of compensation are then studied in a generalized way, assuming that the compensation is uniformly *distributed* along a single transmission line. Later, in Sections 2.4, 2.5, and 2.6, detailed attention is given to *lumped* shunt, series, and dynamic shunt compensation respectively. The effects of each of these types of compensation on voltage control, reactive power requirements, and the steady-state stability limit are systematically explored.

2.1.2. Fundamental Requirements in ac Power Transmission

Bulk transmission of electrical power by ac is possible only if the following two fundamental reqirements are satisfied:

1. *Major synchronous machines must remain stably in synchronism.*

The *major* synchronous machines in a transmission system are the generators and synchronous condensers, all of which are incapable of operating usefully other than in synchronism with all the others.†

The central concept in the maintenance of synchronism is *stability.* Stability is the tendency of the power system (and of the synchronous machines in particular) to continue to operate steadily in the intended mode.‡ It is also a measure of the inherent ability of the system to recover from extraneous disturbances (such as faults, lightning, and changes of load), as well as from planned disturbances (such as switching operations).

One of the limits to the utilization of a transmission line is that for a given length of line the stability tends to become less as the transmitted power is increased. If the power could be gradually increased (with no extraneous disturbances), a level would be reached at which the system

† Synchronous motors are usually (but not always) smaller than even the smallest generators. They are not usually considered individually in the study and planning of the bulk transmission system, even though they can cause severe local disturbances as a result of loss of synchronism. Sometimes the major synchronous machines in one power station or in one region are grouped together and treated as one machine, in order to simplify analysis of the system as a whole. Each group is called a *dynamic equivalent* group, and must remain in synchronism with all the other connected groups.

‡ The intended or normal mode is that in which power and reactive power flows have their intended values, while voltages and currents and the mechanical phase angles between synchronous-machine rotors are all constant.

would suddently become unstable. The synchronous machines at the two ends of the line would pull out of step, that is, lose synchronism. This level of power transmission is the *steady-state stability limit,* so called because it is the maximum steady power that can (in theory) be transmitted stably. The steady-state stability limit is not a hard and fast number fixed forever by the design of the synchronous machines and the transmission equipment. It can be considerably modified by several factors. Among the most important are the excitation of the synchronous machines (and therefore the line voltage); the number and connection of the transmission lines; the number and types of synchronous machines connected (which frequently change with the time of day); the pattern of real and reactive power flows in the system; and, of central interest here, the connection and characteristics of compensation equipment.

It is not practical to operate a transmission system too near to its steady-state stability limit; there must be a margin in the power transfer to allow for disturbances (such as load changes, faults, and switching operations). In determining an appropriate margin, the concepts of *transient* and *dynamic stability* are useful. A transmission system is said to be *dynamically* stable if it recovers normal operation following a specified *minor* disturbance. The degree of dynamic stability can be expressed in terms of the rate of damping of the transient components of voltages, currents, and the load angles of the synchronous machines. The rate of damping, or settling, is the principal interest in a dynamic stability study. Accordingly, modern calculations are usually based on small-perturbation theory and eigenanalysis.

A third major concern in the stability of power transmission systems is whether the system will recover normal operation following a major disturbance, such as a fault severe enough to trip a major circuit, or failure of a major item of plant, such as a generator, overhead line, or transformer. This is the so-called question of *transient stability.* A system has transient stability if it can recover normal operation following a specified *major* disturbance. Whether recovery is possible depends, among other factors, on the level of power transmission immediately before the disturbance occurred. The *transient stability limit* is the highest level of prior power transmission for which the system has transient stability following the specified disturbance.†

2. *Voltages must be kept near to their rated values.*

The second fundamental requirement in ac power transmission is the maintenance of correct voltage levels. Modern power systems are not very tolerant of abnormal voltages, even for short periods.

† For a fuller discussion of these aspects of power system stability, see References 7 through 9.

Undervoltage, which is generally associated with heavy loading and/or a shortage of generation, causes degradation in the performance of loads, particularly induction motors. In heavily loaded systems, undervoltage may be an indication that the load is approaching the steady-state stability limit. Sudden undervoltages can result from the connection of very large loads.

Overvoltage is a dangerous condition because of the risk of flashover or the breakdown of insulation. Saturation of transformers subjected to overvoltage can produce high currents rich in harmonics, and in the presence of sufficient capacitance there is a risk of ferroresonance as well as of harmonic resonances. Overvoltages arise from several causes. The reduction of load during certain parts of the daily load cycle causes a gradual voltage rise. Uncontrolled, this overvoltage would shorten the useful life of insulation even if the breakdown level were not reached. Sudden overvoltage can result from disconnection of loads or other equipment, while overvoltages of extreme rapidity and severity can be caused by line switching operations, faults, and lightning. In long-distance transmission systems, the Ferranti effect (overvoltage at light load) would limit the power transfer and the transmission distance if no compensating measures were taken.

2.1.3. Engineering Factors Affecting Stability and Voltage Control

The design of virtually every item of plant in a transmission system has a bearing on at least one of the fundamental requirements discussed above. The general study of power system voltage control and stability is too vast a subject for this chapter, which is concerned only with compensation techniques. In view of this, the broad picture of power system voltage control and stability is briefly summarized in Tables 1 and 2. (See also the References at the end of this chapter).

In Table 1 the main problems or applications of compensating equipment are grouped under the two fundamental transmission requirements discussed previously, and it can be seen that most of the special-purpose compensating equipments have a role to play under several headings. This makes the general subject of the deployment of compensating equipment a rather complicated one, and the literature is correspondingly extensive. In this chapter emphasis is laid on the theory of what can be achieved using (mainly) series capacitors, shunt capacitors and reactors, polyphase saturated reactors, and thyristor-controlled compensators. In the next section the basic compensation requirements are defined by looking at the *uncompensated* transmission line.

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	PROBLEM, APPLICATION, OR PURPOSE	EXISTING EQUIPMENT OR METHOD: Increase transmission voltage	Increase No. of lines in parallel	Transformer tapchanging	Slow AVR control	Fast AVR control	Fast turbine valving	Rapid line-switching operations, reclosing of circuit-breakers	Braking resistors	SPECIAL-PURPOSE COMPENSATING EQUIPMENT: Shunt reactor (switched/unswitched; linear/nonlinear)	Shunt capacitor	Series reactor	Series capacitor	Synchronous condenser	Polyphase saturated reactor*	Thyristor controlled reactor*	Thyristor switched capacitor	Short-circuit limiting coupling (or Fault current limiter)
FUNDAMENTAL REQUIREMENT #1	Improve steady-state stability	●	●		●	●				●	●		●	●	●	●	●	
	Improve dynamic stability					●								●		●	●	
	Improve transient stability	●				●	●	●	●		●		●	●	●	●	●	
FUNDAMENTAL REQUIREMENT #2	Limit rapid voltage decline					●		●			●			●	●	●	●	
	Limit slow voltage decline			●	●						●			●	●	●	●	
	Limit rapid voltage increase					●		●		●				●	●	●	●	
	Limit slow voltage increase			●	●					●				●	●	●	●	
	Limit fast-wavefront overvoltages due to lightning, switching, etc.									●				●	●	●		
OTHER REQUIREMENTS	Reactive power support at dc converter terminals										●			●	●	●	●	
	Increase short-circuit level												●	●				
	Decrease short-circuit level											●						●

*in conjunction with shunt capacitor where necessary

TABLE 2

Advantages and Disadvantages of Different Types of Compensating Equipment for Transmission Systems

Compensating Equipment	Advantages	Disadvantages
Switched shunt reactor	Simple in principle and construction	Fixed in value
Switched shunt capacitor	Simple in principle and construction	Fixed in value Switching transients
Series capacitor	Simple in principle Performance relatively insensitive to location	Requires overvoltage protection and subharmonic filters Limited overload capability
Synchronous condenser	Has useful overload capability Fully controllable Low harmonics	High maintenance requirement Slow control response Performance sensitive to location Requires strong foundations
Polyphase saturated reactor[a]	Very rugged construction Large overload capability No effect on fault level Low harmonics	Essentially fixed in value Performance sensitive to location Noisy
Thyristor-controlled reactor[a] (TCR)	Fast response Fully controllable No effect on fault level Can be rapidly repaired after failures	Generates harmonics Performance sensitive to location
Thyristor-switched capacitor (TSC)	Can be rapidly repaired after failures No harmonics	No inherent absorbing capability to limit overvoltages Complex buswork and controls Low frequency resonances with system Performance sensitive to location

[a] With shunt capacitors where necessary.

2.2. UNCOMPENSATED TRANSMISSION LINES

2.2.1. Electrical Parameters

A transmission line is characterized by four *distributed* circuit parameters: its series resistance r and inductance l, and its shunt conductance g and capacitance c, the lower-case symbols indicating per-mile values. All four parameters are functions of the line design, that is, of the conductor size, type, spacing, height above ground, frequency, and temperature. They also vary according to the number of nearby parallel lines, and different values are obtained for positive-sequence and zero-sequence currents. Some typical values are given in Table 3.

The characteristic behavior of the line is dominated by the series inductance and the shunt capacitance. Series resistance has a secondary but not insignificant influence, and has a separate importance in determining losses. In this chapter it is largely ignored. Positive-sequence nominal values are assumed, and shunt conductance is ignored. Balanced conditions are assumed except where stated, and one phase of the positive-sequence equivalent circuit is used.

Figure 1 shows a lumped-parameter equivalent circuit of one phase of a transmission line, having identical synchronous machines connected at both ends. Such a line is called *symmetrical.*

2.2.2. Fundamental Transmission Line Equation

The fundamental equation governing the propagation of energy along a transmission line is the wave equation

$$\frac{d^2\mathbf{V}}{dx^2} = \Gamma^2\mathbf{V} \text{ with } \Gamma^2 = (r + j\omega l)(g + j\omega c). \tag{1}$$

Frequency is assumed fixed, and $\mathbf{V}$ is the phasor voltage $\hat{v}e^{j\omega t}/\sqrt{2}$ at any point on the line. The phasor current $\mathbf{I}$ satisfies the equation also. (For derivation see Reference 10). Since x is distance along the line measured from any convenient reference point, the equation describes the variation of the voltage $\mathbf{V}$ and the line current $\mathbf{I}$ along the line, and it implies that both will have a wavelike or sinusoidal variation.

Solution of the transmission line wave equation: Standing Waves. If the line is assumed lossless, the general solution of Equation 1 (for voltage and current) is

$$\mathbf{V}(x) = \mathbf{V}_r \cos\beta(a - x) + jZ_0\mathbf{I}_r \sin\beta(a - x) \tag{2a}$$

TABLE 3
Typical EHV-UHV Transmission Line Parameters[a,b]

	Nominal Voltage (kV)									
	345		500			765		1100		1500
	Conductor Configuration									
Characteristics	Hor.	Delta	Hor.	Vert.	Delta	Hor.	Delta	Hor.	Delta	Hor.
ωl (Ω/mi)	0.59	0.59	0.60	0.49	0.51	0.53	0.53	0.48	0.47	0.47
r (Ω/mi)	0.060	0.060	0.025	0.026	0.019	0.020	0.020	0.0079	0.0076	0.0072
ωc (μS/mi)	7.27	7.29	7.23	8.80	8.35	8.02	8.01	8.94	8.92	9.21
β (rad/mi $\times 10^{-3}$)	2.07	2.07	2.08	2.08	2.06	2.06	2.06	2.07	2.05	2.08
Surge impedance Z_0 (Ω)	285	283	287	235	247	258	257	232	231	225
Natural load P_0 (MW)	417	420	870	1060	1010	2270	2280	5220	5250	10000
Charging MVA (MVA/mi)	0.866	0.868	1.81	2.20	2.09	4.70	4.70	10.8	10.8	20.7
Line current at SIL (A)	700	700	1000	1230	1170	1710	1720	2740	2750	3850

[a] Reproduced by kind permission of J.J. LaForest, Electric Utility Systems Engineering Department.

[b] This table contains typical positive-sequence characteristics of transmission lines from 345 to 1500 kV, calculated at nominal voltage, 60 Hz.

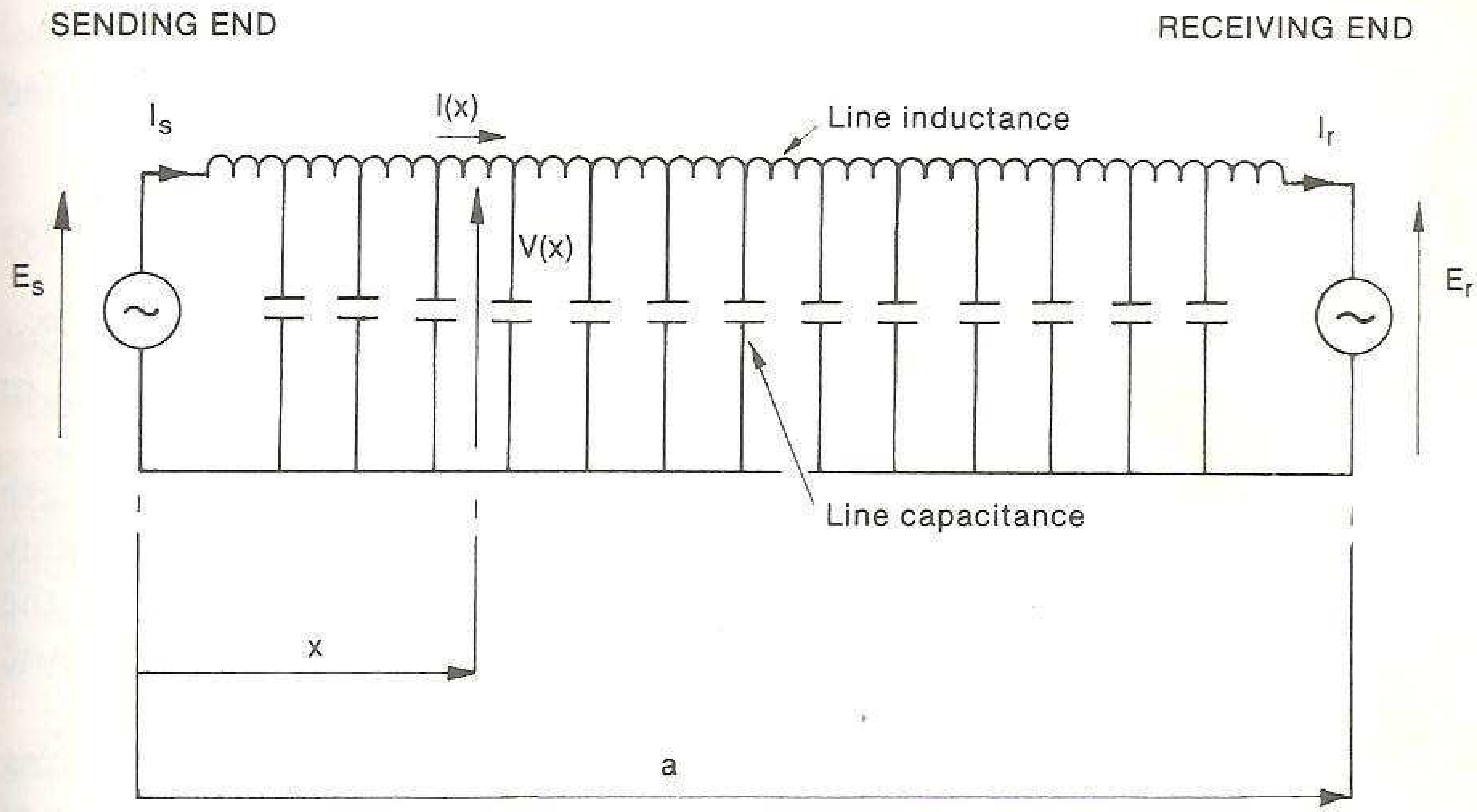


FIGURE 1. Lumped-element representation of a long transmission line.

$$\mathbf{I}(x) = \mathrm{j}\left(\frac{\mathbf{V}_r}{Z_0}\right) \sin\beta\,(a - x) + \mathbf{I}_r \cos\beta\,(a - x)\,, \tag{2b}$$

where β is derived from the propagation constant Γ by putting $r = g = 0$. This gives $\Gamma = \mathrm{j}\beta$ and

$$\beta = \omega\sqrt{lc}\,. \tag{3}$$

The form of Equation 2 shows the expected sine wave variation of **V** and **I** along the line, each quantity having two terms or components. **V** and **I** are said to form *standing waves* because of the sinusoidal variation of both their real and imaginary parts along the line.

The quantity $1/\sqrt{lc}$ is the propagation velocity of electromagnetic effects along the line. For overhead high-voltage transmission lines it has a value somewhat less than the velocity of light, $u = 3 \times 10^8$ m/sec = 186,000 mi/sec. Since also $\omega = 2\pi f$, Equation 3 gives

$$\beta = \frac{2\pi f}{u} = \frac{2\pi}{\lambda}\,, \tag{4}$$

where λ is the wavelength. β is the wave number, that is, the number of complete waves per unit of line length.

At 60 Hz λ = 3100 mi and β can be expressed as one wavelength per 3100 mi, that is, 360° per 3100 mi, or 0.116° per mi, or 2.027×10^{-3} rad/mi. The quantity βa is the *electrical length* of the line expressed in radians or in wavelengths: symbol θ.

2.2.3. Surge Impedance and Natural Loading

The constant Z_0 in Equation 2 is the *surge impedance* (sometimes called the characteristic impedance):

$$Z_0 = \sqrt{\frac{l}{c}}. \tag{5}$$

Its value depends on the line design (see Section 2.2.1 and Table 3). For high-voltage overhead lines, the positive-sequence value typically lies in the range 200–400 Ω.

If losses are neglected the line is characterized entirely by its length and by the two parameters Z_0 and β. Since these values are roughly comparable for all lines, the behavior of all lines is fundamentally the same, and differences only arise according to the length, the voltage, and the level of power transmission.

The surge impedance is the apparent impedance of an infinitely long line, that is, the ratio of voltage to current at any point along it. A line of finite length terminated at one end by an impedance Z_0 is electrically indistinguishable from an infinite line, so that if $\mathbf{V}_r/\mathbf{I}_r = Z_0$ then from Equation 2 the apparent impedance at any point is

$$\mathbf{Z}(x) = \frac{\mathbf{V}(x)}{\mathbf{I}(x)} = \frac{Z_0\mathbf{I}_r[\cos\beta(a-x) + j\sin\beta(a-x)]}{\mathbf{I}_r[\cos\beta(a-x) + j\sin\beta(a-x)]} = Z_0 \tag{6}$$

which is independent of x. More importantly,

$$\mathbf{V}(x) = \mathbf{V}_r[\cos\beta(a-x) + j\sin\beta(a-x)] = \mathbf{V}_r e^{j\beta(a-x)} \tag{7a}$$

$$\mathbf{I}(x) = \mathbf{I}_r[\cos\beta(a-x) + j\sin\beta(a-x)] = \mathbf{I}_r e^{j\beta(a-x)}, \tag{7b}$$

that is, both $\mathbf{V}$ and $\mathbf{I}$ have constant amplitude along the line. The line is said to have a *flat voltage profile.* While $\mathbf{V}$ and $\mathbf{I}$ are in phase with each other all along the line, both are rotated in phase. The phase angle between sending-end and receiving-end quantities is implicit in Equation 7; it is $\theta = \beta a$ rad. For a 200-mi line at 60 Hz the angle is 0.405 rad or 23.2°. The phasor relationships are shown in Figure 2.

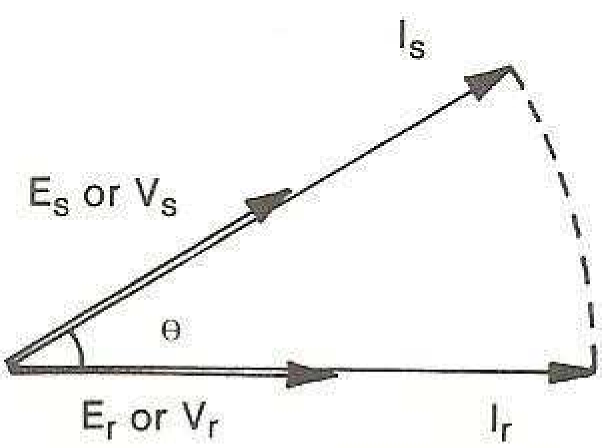


FIGURE 2. Phasor diagram of naturally loaded line.

A line in this condition is said to be *naturally loaded.* The natural load (or *surge-impedance load,* SIL) is

$$P_0 = \frac{V_0^2}{Z_0}, \tag{8}$$

where V_0 is the nominal or rated voltage of the line. If V_0 is the line-to-neutral voltage, Equation 8 gives the per-phase value of surge-impedance power; if V_0 is the line-to-line voltage, P_0 is the three-phase value. The natural load is an important reference quantity which will be used extensively below.

An advantage of operating the line at the natural load is that because of the flat voltage profile, the insulation is uniformly stressed at all points.

The natural load of the uncompensated line increases with the *square* of the voltage (Equation 8). This helps to explain why transmission voltages have increased as the level of transmitted power has grown. Table 3 shows the natural load for some common line voltages.

The surge impedance Z_0 is a real number. Therefore, at the natural load the power factor — that is, the cosine of the angle between **V** and **I** — is unity at all points along the line, including the ends. This is apparent from Equation 6. It means that at the natural load *no reactive power has to be absorbed or generated at either end.* The reactive power generated in the shunt capacitance of the line is exactly absorbed by the series inductance. This important condition can be further explained as follows. In any short element of the line the reactive power per unit length generated by the shunt capacitance is $V^2 b = V^2 \omega c$, while the reactive power per unit length absorbed by the series inductance is $I^2 \omega l$. For reactive-power balance in this element of line,

$$V^2 \omega c = I^2 \omega l,$$

i.e.,

$$\frac{V}{I} = \sqrt{\frac{l}{c}} = Z_0. \tag{9}$$

This must be true at all points along the line, including the sending and the receiving end. Therefore, reactive power balance is achieved at the natural loading, with $P_0 = V^2/Z_0$. This is the only value of transmitted power that gives a flat voltage profile and unity power factor at both ends of the line.

In the sense that P_0 is the natural *power* of the line, the "natural" *reactive* power is zero.

2.2.4. The Uncompensated Line on Open-Circuit

Voltage and Current Profiles. A lossless line that is energized by generators at the sending end and is open-circuited at the receiving end is described by Equations 2a and b with $\mathbf{I}_r = 0$, so that

$$\mathbf{V}(x) = \mathbf{V}_r \cos\beta(a - x) \tag{10a}$$

and

$$\mathbf{I}(x) = \mathrm{j}\left(\frac{\mathbf{V}_r}{Z_0}\right) \sin\beta(a - x)\ . \tag{10b}$$

The voltage and current at the sending end are given by these equations with $x = 0$:

$$\mathbf{E}_s = \mathbf{V}_r \cos\theta\ ; \tag{11a}$$

$$\mathbf{I}_s = \mathrm{j}\left(\frac{\mathbf{V}_r}{Z_0}\right) \sin\theta = \mathrm{j}\left(\frac{\mathbf{E}_s}{Z_0}\right) \tan\theta\ . \tag{11b}$$

$\mathbf{E}_s$ and $\mathbf{V}_r$ are in phase, which is consistent with the fact that there is no power transfer. (See Section 2.2.6). The phasor diagram is shown in Figure 3.

The line *voltage profile* expressed by Equation 10a can be written more conveniently in terms of $\mathbf{E}_s$:

$$\mathbf{V}(x) = \mathbf{E}_s \frac{\cos\beta(a - x)}{\cos\theta}\ . \tag{12a}$$

Similarly the *current profile* is given by

$$\mathbf{I}(x) = \mathrm{j}\,\frac{\mathbf{E}_s}{Z_0}\,\frac{\sin\beta(a - x)}{\cos\theta}\ . \tag{12b}$$

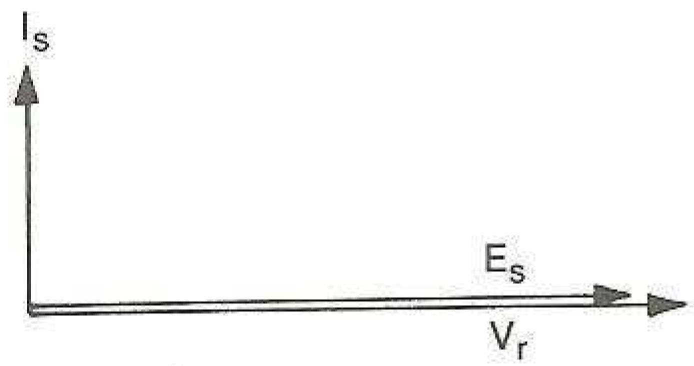


FIGURE 3. Phasor diagram of 200-mi line open-circuit at the receiving end.

These profiles are shown in Figure 4 for a line 200 miles in length, for which at 60 Hz $\theta = 0.405$ radian $= 23.2°$. With $E_s = 1.0$ pu the receiving-end voltage is $V_r = 1.088$ pu, that is a rise of 8.8%. This rise is called the *Ferranti effect.*

A rise of 8.8% is not enough to cause severe problems for insulation or for voltage regulating equipment. But at 400 mi the open-circuit voltage would be 1.579 pu which is unacceptable, if not dangerous. At 775 mi (one quarter-wavelength) the voltage rise would be infinite; operation of such a line is completely impractical without some means of compensation.

In practice the open-circuit voltage rise will be greater than is indicated by Equation 11a, which assumes that the sending-end voltage is fixed. Following a sudden open-circuiting of the line at the receiving end, the sending-end voltage tends to rise immediately to the *open-circuit voltage* of the sending-end generators, which exceeds the terminal voltage by approximately the voltage drop due to the prior current flowing in their short-circuit reactances. In spite of its practical importance, this complication will not be considered further, except to note that typically it is desirable to limit the open-circuit voltage rise to about 25% at the sending end and about 40% at the receiving end under worst conditions—that is,

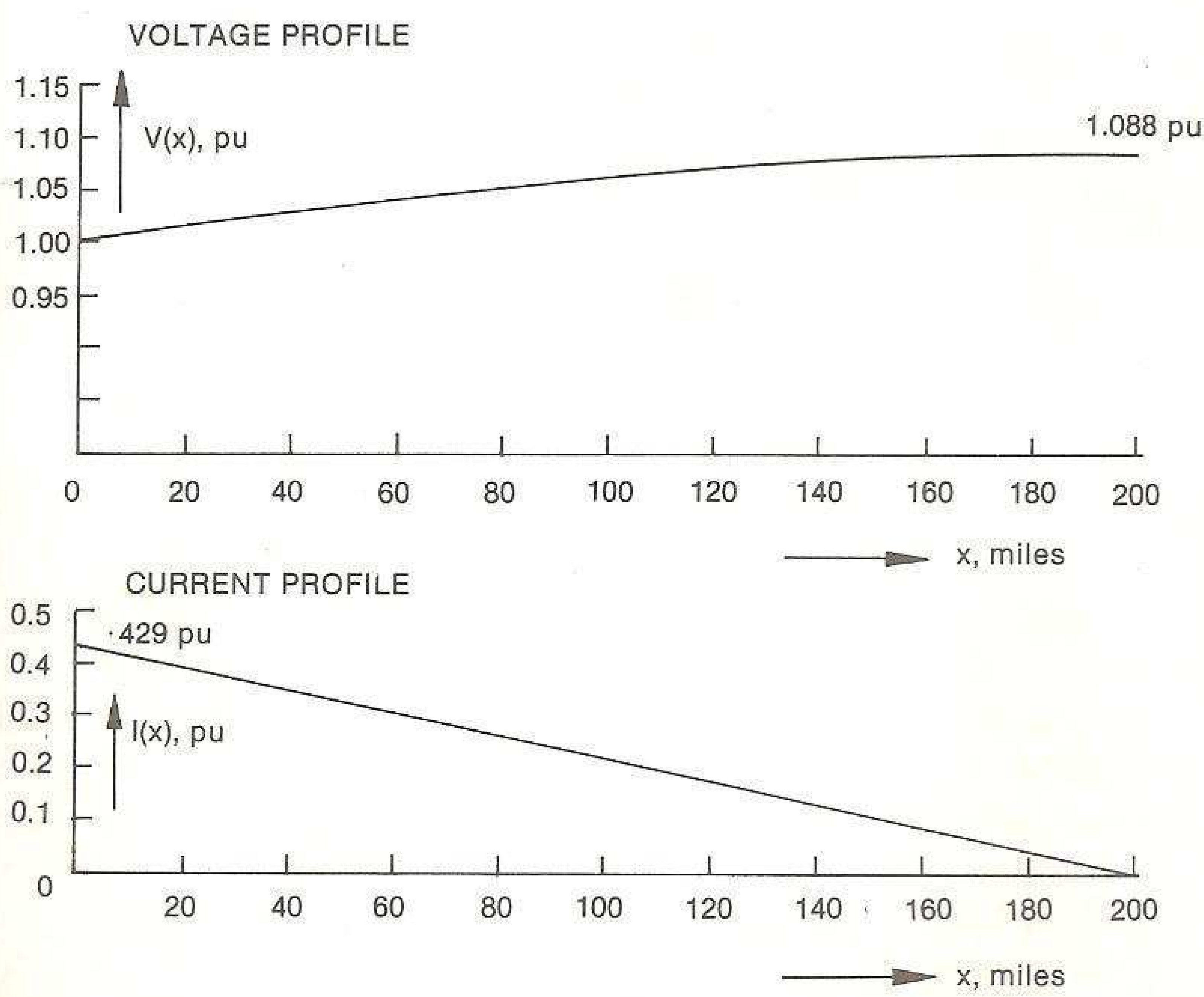


FIGURE 4. Voltage and current profiles for a 200-mi line open-circuited at the receiving end.

with all parallel lines connected, minimum generators connected, and before the excitation on the generators has been reduced to bring the voltage down to a safe level.

The magnitude of $\mathbf{I}_s$ in Figure 3 is 0.429 pu. This means that the *line-charging current* flowing in the sending-end generators is 42.9% of the current corresponding to the natural load.

The Symmetrical Line at No-Load. Akin to the open-circuited line energized from one end is the symmetrical line at no load. This is a line with identical synchronous machines at both ends, but no power transfer. Suppose that the terminal voltages are controlled to have the same *magnitude,* that is, $E_s = E_r$. From Equations 2a and b, with $x = 0$,

$$\mathbf{E}_s = \mathbf{E}_r \cos\theta + jZ_0\mathbf{I}_r \sin\theta \; ; \tag{13a}$$

$$\mathbf{I}_s = j\left(\frac{\mathbf{E}_r}{Z_0}\right) \sin\theta + \mathbf{I}_r \cos\theta \; . \tag{13b}$$

With no power transfer the electrical conditions are the same at both ends. Therefore by *symmetry*†

$$\mathbf{I}_s = -\mathbf{I}_r \; . \tag{14}$$

From Equation 13b,

$$-\mathbf{I}_r = j\,\frac{\mathbf{E}_r}{Z_0}\,\frac{\sin\theta}{1+\cos\theta} = j\frac{\mathbf{E}_r}{Z_0}\tan\frac{\theta}{2} \; . \tag{15}$$

Substituting this value for $\mathbf{I}_r$ in Equation 13a gives

$$\mathbf{E}_s = \mathbf{E}_r \tag{16}$$

and therefore

$$\mathbf{I}_s = j\,\frac{\mathbf{E}_s}{Z_0}\tan\frac{\theta}{2} \; . \tag{17}$$

Equation 16 shows that $\mathbf{E}_r$ and $\mathbf{E}_s$ are in phase, which again is consistent with the fact that there is no power transfer. The current at each end is line-charging current. Comparison of Equations 15 and 17 with Equation 11b shows that the line is equivalent to two equal halves connected back-to-back. Half the line-charging current is supplied from each end.[(1)] The phasor diagram is shown in Figure 5 for $a = 200$ mi, with $E_s = E_r = V_0 = 1.0$ pu.

† The negative sign in Equation 14 arises because of the convention in which positive current flows away from the sending end and towards the receiving end.

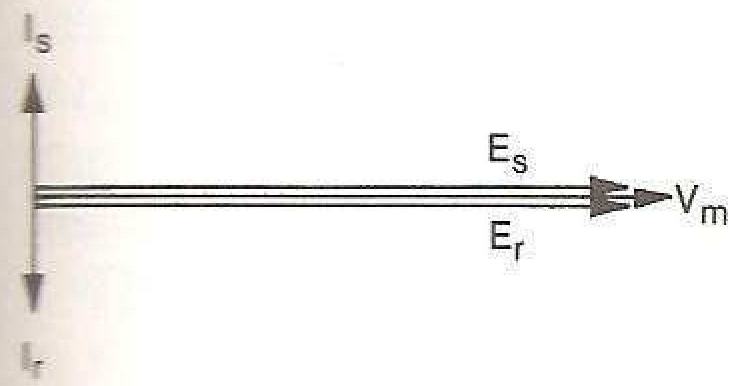


FIGURE 5. Phasor diagram of 200-mi symmetrical line.

By symmetry the midpoint current is zero. The midpoint voltage is, therefore, equal to the open-circuit voltage of a line having half the total length:

$$V_m = \frac{E_s}{\cos(\theta/2)} \, . \tag{18}$$

The voltage and current profiles for the symmetrical line at no load can be derived from Equations 12a and b with a replaced by $a/2$:

$$\mathbf{V}(x) = \mathbf{E}_s \frac{\cos\beta(a/2 - x)}{\cos(\theta/2)} \tag{19a}$$

and

$$\mathbf{I}(x) = \mathrm{j}\,\frac{\mathbf{E}_s}{Z_0}\,\frac{\sin\beta(a/2 - x)}{\cos(\theta/2)} \tag{19b}$$

for $x \leqslant a/2$. For the other half of the line, that is, $a/2 \leqslant x \leqslant a$,

$$\mathbf{V}(x) = \mathbf{V}(a - x) \tag{19c}$$

and

$$\mathbf{I}(x) = -\,\mathbf{I}(a - x) \, . \tag{19d}$$

The profiles are shown in Figure 6. It is interesting to compare these with Figure 4.

If $E_s \neq E_r$ the current and voltage profiles are no longer symmetrical and the highest voltage is no longer at the midpoint, but is nearer to the end of the line which has the higher terminal voltage. The currents in the synchronous machines are also unequal.

Underexcited Operation of Generators Due to Line-Charging. With $\mathbf{I}_r = 0$ the charging reactive power at the sending end is given by

$$Q_s = \mathrm{Im}\,(\mathbf{E}_s \mathbf{I}_s^*) \, . \tag{20}$$

The line-charging current is given by Equation 11b, so that if E_s is equal to the rated voltage of the line,

$$Q_s = -\,P_0 \tan\theta \, . \tag{21}$$

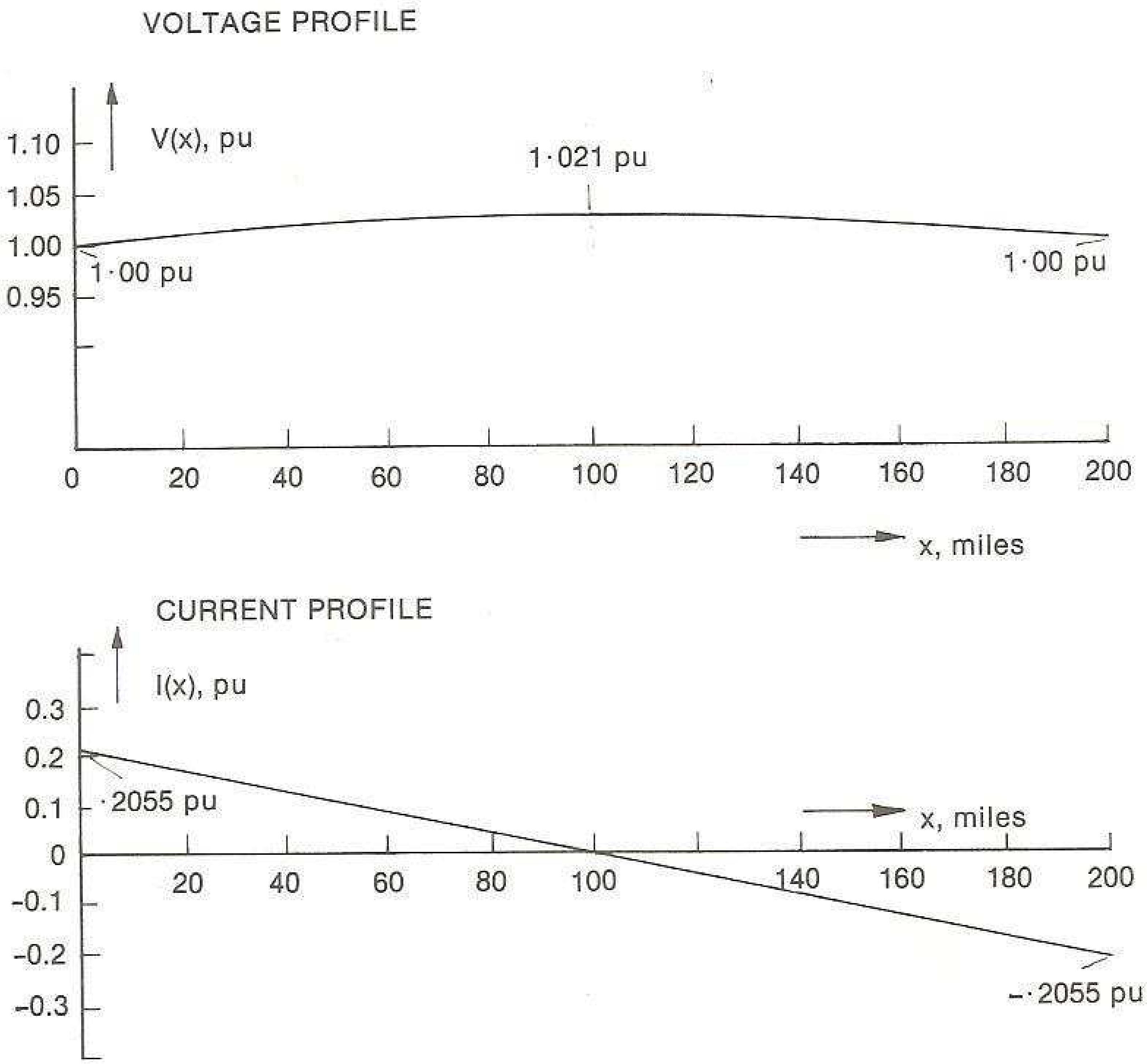


FIGURE 6. Voltage and current profiles for a 200-mi symmetrical line.

The charging current leads the line terminal voltage by 90° and flows in the generators. For the 200-mi line $I_s = 0.429$ pu and so Q_s is nearly 43% of the natural load expressed in MVA. At 400 kV the generators would have to absorb 172 MVAr.

The reactive power absorption capability of synchronous generators is limited for two reasons. First, underexcited operation increases the heating of the ends of the stator core. Second, the reduced field current reduces the internal emf of the generators, and this impairs stability (see later). The absorption limit is typically not more than 0.45 pu of the MVA rating. In the 200-mi example line, if the total MVA rating of the synchronous generators is equal to the natural load, the charging reactive power at 0.43 pu would be just within the limit. But if (for economy) half the generators were disconnected, the load being small or zero, the remainder would have to absorb 0.86 pu of their MVA capacity, which is definitely above the limit.

Aside from using compensation, there are two main ways in which this problem can sometimes be alleviated. First, if the line is made up of two or more parallel circuits, one or more of the circuits can be switched off under light-load or open-circuit conditions. This is permissible only if the consequent reduction in security of supply at the receiving end is acceptable. Second, if the generator absorption is limited by stability and not by core-end heating, the absorption limit can be increased by using a rapid-response excitation system which restores the stability margins when the steady-state field current is low.

The underexcited operation of generators can set a more stringent limit to the maximum length of an uncompensated line than the open-circuit voltage rise. Let the total generator rating be P_g and let their maximum reactive power absorption be $q_u P_g$. This must not be less than the line-charging reactive power given by Equation 21. It follows that the generating capacity must satisfy the relation

$$P_g \geqslant \frac{P_0 \tan \theta}{q_u} . \tag{22}$$

If, for instance, $q_u = 0.3$ and the sending-end generating capacity is $P_g = P_0$, the maximum length of uncompensated line is only 144 mi. Alternatively a line 200 mi long would require $P_g \geqslant 1.43\ P_0$ if q_u were limited to 0.3. It would generally be wasteful to have so much excess generating capacity connected (or even installed) merely in order to satisfy the line-charging requirement. It is better to satisfy this requirement by means of compensation. Shunt reactors, synchronous condensers, or static compensators can be connected at the receiving end or at points along the line. Their ratings and points of connection should ideally be optimized and coordinated with other equipment to achieve satisfactory control of line voltage under all conditions, as well as to relieve the generators of excessive reactive power absorption.

2.2.5. The Uncompensated Line Under Load: Effect of Line Length, Load Power, and Power Factor on Voltage and Reactive Power

Radial Line with Fixed Sending-end Voltage. A load $P + jQ$ at the receiving end of a transmission line draws the current

$$\mathbf{I}_r = \frac{P - jQ}{\mathbf{V}_r} . \tag{23}$$

From Equation 2a with $x = 0$, if the line is assumed lossless the sending- and receiving-end voltages are related by

$$\mathbf{E}_s = \mathbf{V}_r \cos \theta + jZ_0 \sin \theta \, \frac{P - jQ}{\mathbf{V}_r} . \tag{24}$$

If $\mathbf{E}_s$ is fixed, this quadratic equation can be solved for $\mathbf{V}_r$. The solution shows how $\mathbf{V}_r$ varies with the load and its power factor and with the line length. A typical result is shown in Figure 7, for which $a = 200$ mi. The magnitude V_r is plotted against the normalized load power P/P_0 for five different power factors, with $E_s = V_0 = 1.0$ pu.

Several fundamentally important properties of ac transmission are evident from Figure 7. For each load power factor there is a *maximum transmissible power.* (See Section 2.5). For any value of P below the maximum there are two possible solutions for V_r (i.e., two roots of Equation 24). Normal operation of the power system is always at the upper value, within narrow limits around 1.0 pu. When $P = Q = 0$, Equation 24 reduces to Equation 11a for the open-circuit condition. Also apparent in Figure 7 is the flat voltage profile achieved at unity power factor when $P = P_0$, that is, $V_r = E_s$.

The load power factor has a strong influence on the receiving-end voltage. Loads with lagging power factor, with unity, or with very high leading power factor, tend to reduce V_r as the load P increases. With leading power factors (except those very near unity), the tendency is to increase V_r until P reaches a much higher value. Leading power factor loads generate reactive power which supplements the line-charging reactive power and tends to support the line voltage.

The effect of the line length can be determined by redrawing Figure 7 for different values of a. Figures 8a through c show the results for three different power factors with $a = 100, 200, 300, 400$, and 500 mi. It appears from Figure 8 that uncompensated lines between about 100 and 200 mi long can be operated at normal voltage provided that the load power factor is high. Because of their large voltage variations, longer lines are impractical at all power factors unless some means of voltage

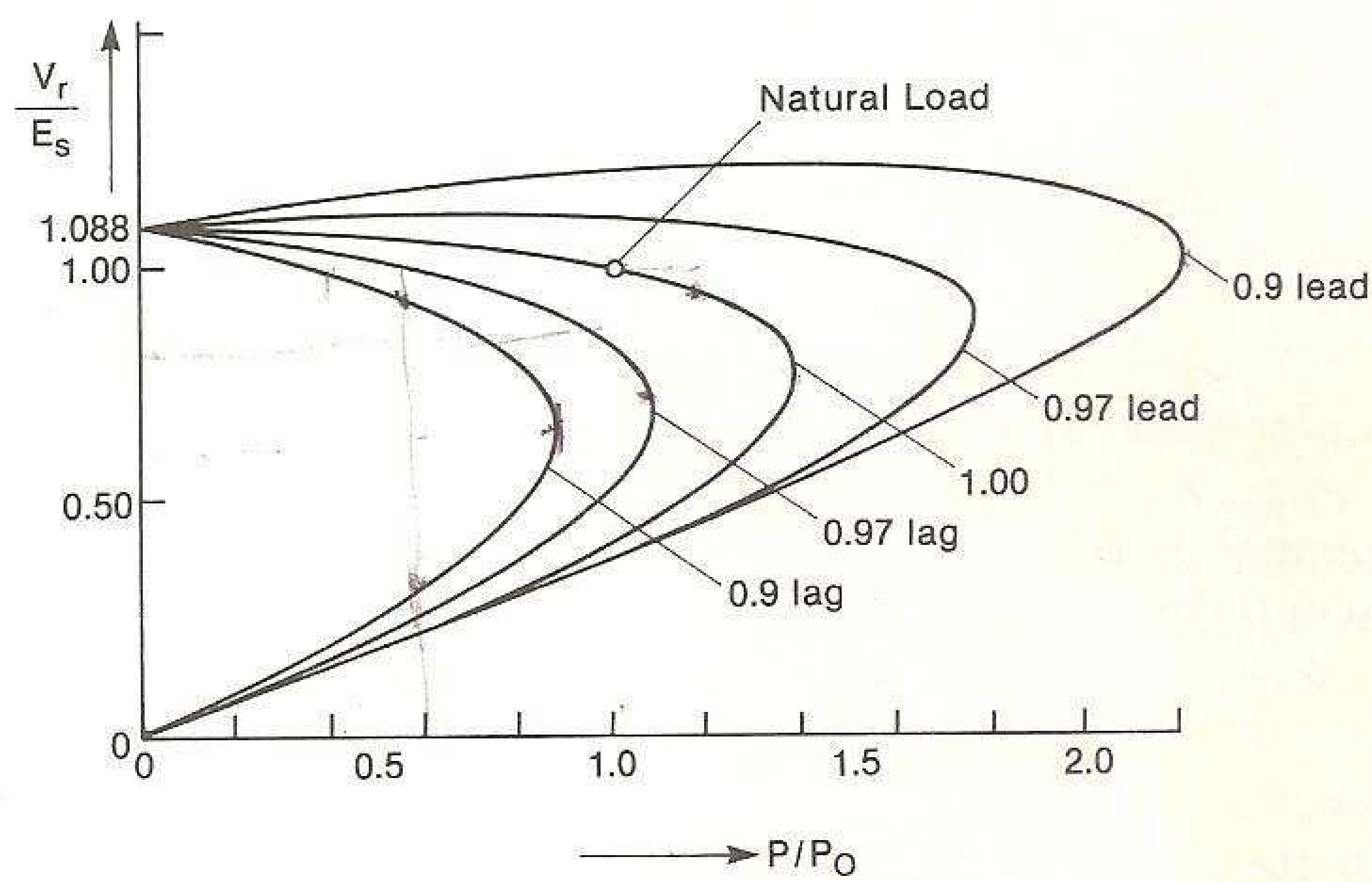


FIGURE 7. Receiving-end voltage magnitude as a function of load (P) and load power factor for a 200-mi lossless radial line.

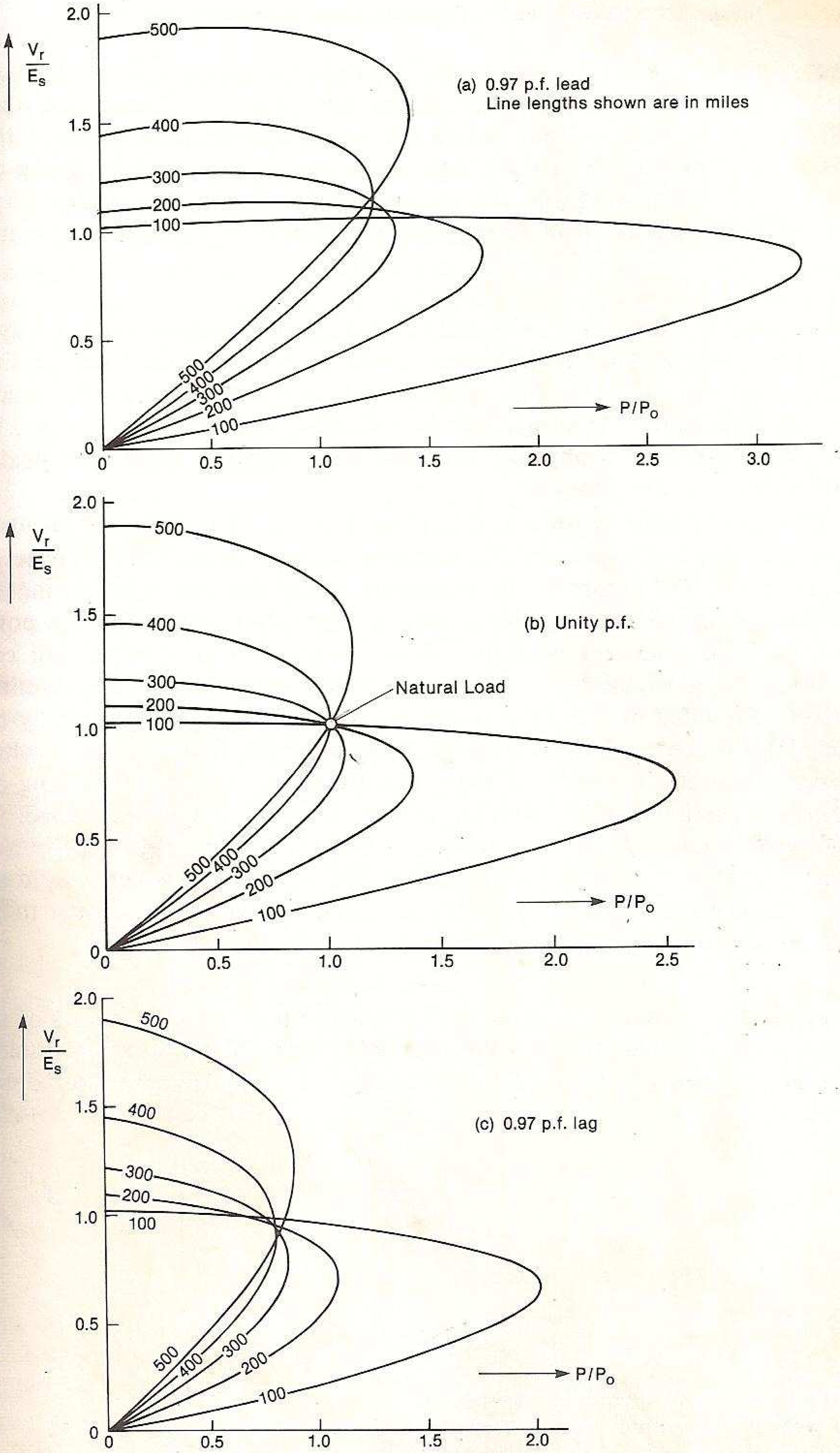


FIGURE 8. Receiving-end voltage as a function of line length, load, and load power factor (radial line).

control or compensation is provided. Even though $V_r = E_s = 1.0$ pu at the natural load ($P + jQ = P_0$), if the line length is longer than about 200 mi V_r is extremely sensitive to any *variation* in P. If a is greater than 390 mi or $\lambda/8$ (i.e., $\theta > 45°$) then at the natural load the receiving-end voltage is the *lower* of the two roots of Equation 24: See Figure 8*b* for $a = 400$ and 500 mi, with $P/P_0 = 1$. In virtually all cases such operation would be unstable.

Symmetrical Line. In Section 2.2.4 the no-load behavior of the symmetrical line was deduced in terms of two open-circuited half-length lines connected back-to-back. The symmetrical line under load can be treated in the same way. Although the symmetrical line is a special case, the treatment provides a physical understanding which is helpful in dealing with more complex cases.[1]

By definition the symmetrical line has $E_s = E_r$. Under load $\mathbf{E}_s$ leads $\mathbf{E}_r$ in phase, and by symmetry the midpoint voltage is midway in phase between them: See Figure 9. By symmetry again, the power factor angle at one end must be the negative of that at the other end, while the power factor at the midpoint is unity. This being so, it is possible and convenient to use Figure 8*b* to describe how V_m varies with the transmitted power. Provided that $E_s = E_r = 1.0$ pu, the line length is replaced by $a/2$ and V_m can be read off Figure 8*b*. For example, the midpoint voltage variations on a symmetrical 200-mi line are the same as the receiving-end voltage variations on a 100-mi line with a unity power-factor load. At 200 mi a marked improvement results from having synchronous machines at both ends. A symmetrical 500-mi line, however, would still have unacceptably large voltage variations at the midpoint (equal to the receiving-end variations on a 250-mi line).

Reactive Power Requirements. The reactive power requirements of the line are determined by the voltage and the level of power transmission. It is important to know what these requirements are, because they deter-

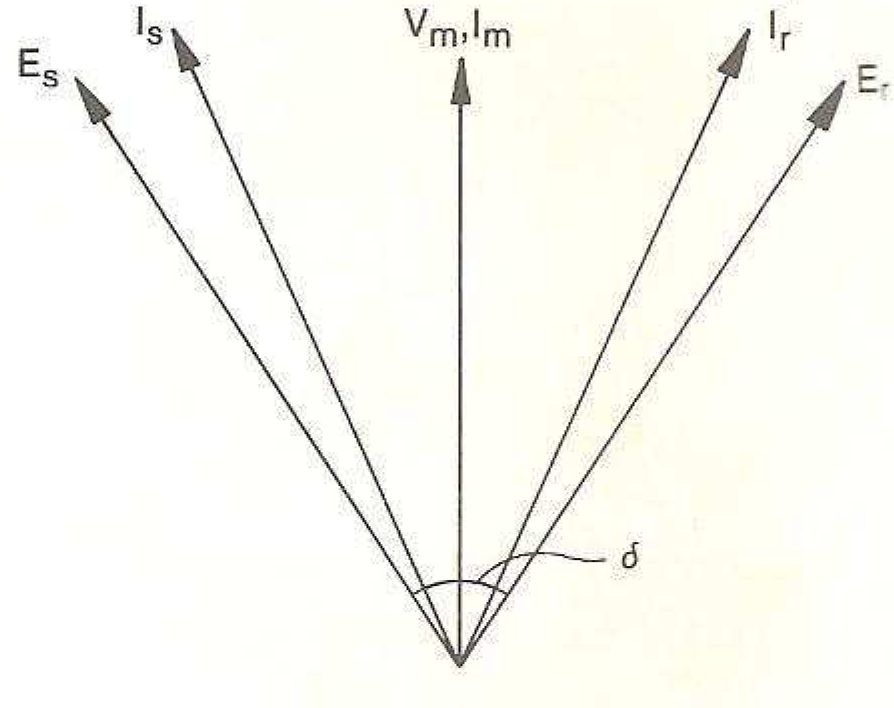


FIGURE 9. Phasor diagram of symmetrical line with $P > P_0$. Note that the receiving end has a leading power factor and that both ends are supplying reactive power to the line.

mine the reactive power ratings of the terminal synchronous machines as well as of any compensating equipment. Note that the terminal power factor is the resultant of all circuits connected at that end of the line. Any inductive load connected, for example, at the sending end will assist the synchronous generators to absorb the line-charging reactive power. In general, in the absence of compensating equipment, the synchronous machines must absorb or generate the difference between the reactive power of the line and that of the local load.

The equations for the sending-end half of the symmetrical line are

$$\mathbf{E}_s = \mathbf{V}_m \cos\frac{\theta}{2} + jZ_0\mathbf{I}_m \sin\frac{\theta}{2} \tag{25a}$$

$$\mathbf{I}_s = j\frac{\mathbf{V}_m}{Z_0}\sin\frac{\theta}{2} + \mathbf{I}_m \cos\frac{\theta}{2}. \tag{25b}$$

At the midpoint,

$$P_m + jQ_m = \mathbf{V}_m\mathbf{I}_m^* = P, \tag{26}$$

where P is the transmitted power. Note that $Q_m = 0$; that is, no reactive power flows past the midpoint. The real and reactive power which must be supplied at the sending end are given by

$$P_s + jQ_s = \mathbf{E}_s\mathbf{I}_s^* . \tag{27}$$

Substituting for $\mathbf{E}_s$ and $\mathbf{I}_s$ from Equation 25 and treating $\mathbf{V}_m$ as reference phasor, $P_m = V_m I_m$ and

$$P_s + jQ_s = P + j\frac{\sin\theta}{2}\left[Z_0 I_m^2 - \frac{V_m^2}{Z_0}\right]. \tag{28}$$

Since the line is assumed lossless, the result $P_s = P$ is expected; likewise $P_r = P$ at the receiving end. The expression for Q_s can be rearranged as follows. Making use of the relations $P_0 = V_0^2/Z_0$ and $P_m = V_m I_m$,

$$Q_s = P_0\frac{\sin\theta}{2}\left[\left(\frac{P}{P_0}\right)^2\left(\frac{V_0}{V_m}\right)^2 - \left(\frac{V_m}{V_0}\right)^2\right]. \tag{29}$$

This equation shows how the midpoint voltage is related to the reactive power requirement of the symmetrical line. By symmetry, Equation 29 applies to both ends of the line, and each end supplies half the total. Because of the reactive power sign convention, this is written $Q_s = -Q_r$.

Where $P = P_0$, that is, at the natural load, if $V_m = 1.0$ pu Equation 29 gives the familiar result: $Q_s = 0$. In this condition $Q_r = 0$ also, and

$E_s = E_r = V_m = V_0 = 1.0$ pu. At no load, that is, $P = 0$, if the terminal voltages are both adjusted so that $E_s = E_r = V_0 = 1.0$ pu, then $I_m = 0$ and from Equation 29.

$$Q_s = -P_0 \tan \frac{\theta}{2} . \tag{30}$$

This is identical in form to Equation 21. It shows that when $E_s = E_r$ and $P = 0$, the sending-end reactive power is the line-charging reactive power for half the line. The receiving-end generators absorb an equal amount from the other half.

If the terminal voltages are continuously adjusted so that the midpoint voltage $V_m = V_0 = 1.0$ pu at all levels of power transmission, then from Equation 29

$$Q_s = P_0 \frac{\sin \theta}{2} \left[\left(\frac{P}{P_0} \right)^2 - 1 \right] = -Q_r . \tag{31}$$

In addition, from Equations 25 and 26 it can be shown that for $V_m = V_0$,

$$E_s = V_m \sqrt{1 - \sin^2 \frac{\theta}{2} \left[1 - \left(\frac{P}{P_0} \right)^2 \right]} = E_r . \tag{32}$$

These two equations illustrate the general behavior of the symmetrical line. If $P < P_0$, the midpoint voltage is higher than the terminal voltages. If $P > P_0$, the reverse is true, and if $P = P_0$ the voltage profile is flat. When $P < P_0$ there is an excess of line-charging reactive power; that is, Q_s is negative and Q_r is positive, indicating absorption at both terminals. When $P > P_0$ there is an overall deficit of reactive power in the line. The excess or deficit can be corrected by means of compensation, as is seen in Section 2.3.

It should be noted that the reactive power requirement is determined by the *square* of the power transmitted. As an example, consider an uncompensated symmetrical line 200 mi long with $P = 1.5\ P_0$. Then $\sin \theta = 0.394$ and $Q_s = -Q_r = 0.246\ P_0$. For every megawatt of transmitted power, a total reactive power of $2 \times 0.246/1.5 = 0.329$ MVAr has to be supplied from the ends.

Alternative useful equations for the reactive power requirements are given later in Section 2.2.6.

2.2.6. The Uncompensated Line Under Load: Maximum Power and Stability Considerations

Symmetrical Line. If the load at the receiving end of a lossless transmission line is $P + jQ$, then the terminal voltages are related by Equation 24:

$$\mathbf{E}_s = \mathbf{E}_r \cos\theta + jZ_0 \frac{P - jQ}{\mathbf{E}_r} \sin\theta \,. \tag{33}$$

This equation is valid for synchronous and nonsynchronous loads alike. Here the load is assumed to be synchronous and $\mathbf{E}_r$ is written instead of $\mathbf{V}_r$. If $\mathbf{E}_r$ is taken as reference phasor, $\mathbf{E}_s$ can be written as

$$\mathbf{E}_s = E_s\, e^{j\delta} = E_s\,(\cos\delta + j\sin\delta)\,, \tag{34}$$

where δ is the phase angle between $\mathbf{E}_s$ and $\mathbf{E}_r$ (see Figure 9). δ is called the *load angle* or the *transmission angle.* Equating the real and imaginary parts of Equations 33 and 34,

$$E_s \cos\delta = E_r \cos\theta + Z_0 \frac{Q}{E_r} \sin\theta \,, \tag{35}$$

$$E_s \sin\delta = Z_0 \frac{P}{E_r} \sin\theta \,. \tag{36}$$

Equation 36 can be rearranged in the form

$$P = \frac{E_s E_r}{Z_0 \sin\theta} \sin\delta \,. \tag{37}$$

This equation is important because of its simplicity and its wide-ranging validity. The equation is true when $E_s \neq E_r$ and is valid for synchronous and nonsynchronous loads alike. Its only major shortcoming is that it neglects losses. A more familiar form is obtained when, for an electrically short line, $\sin\theta$ is replaced by $\theta = \beta a = \omega a\sqrt{lc}$. Then $Z_0\theta = \omega a\sqrt{lc}\,.\ \sqrt{l/c} = \omega a l = X_l$, the series reactance of the line, and

$$P \simeq \frac{E_s E_r}{X_l} \sin\delta \,. \tag{38}$$

Equation 37 shows that if E_s and E_r are fixed, the power transmitted can be expressed in terms of only one variable: the transmission angle δ. If $E_s = E_r = V_0$, then

$$P = \frac{P_0}{\sin \theta} \sin \delta \, . \tag{39}$$

Figure 10 shows this relationship for a 200-mi line at 60 Hz, for which $\theta = 0.405$ rad and $\sin \theta = 0.394$. The graph is ordinarily plotted with P as ordinate; but here it is rotated clockwise by 90° to reflect the fact that P is the independent and δ the dependent variable. It also reflects the similarity with the voltage-versus-power characteristic of Figure 8b, which for convenience is reproduced as Figure 11 with the correct line-lengths for the symmetrical line. It can be seen that as the load is increased from zero the load angle increases. If E_s and E_r are held constant the voltage profile "sags," the midpoint voltage experiencing the greatest decrease along the line.

As indicated in Section 2.2.5 there is a maximum power that can be transmitted. It is useful to have a physical understanding of this phenomenon. Let the sending-end synchronous machines be thought of as an equivalent synchronous generator, and the receiving-end machines as an equivalent synchronous motor. The load angle δ is then a measure of the relative mechanical position of the rotors of these two machines as

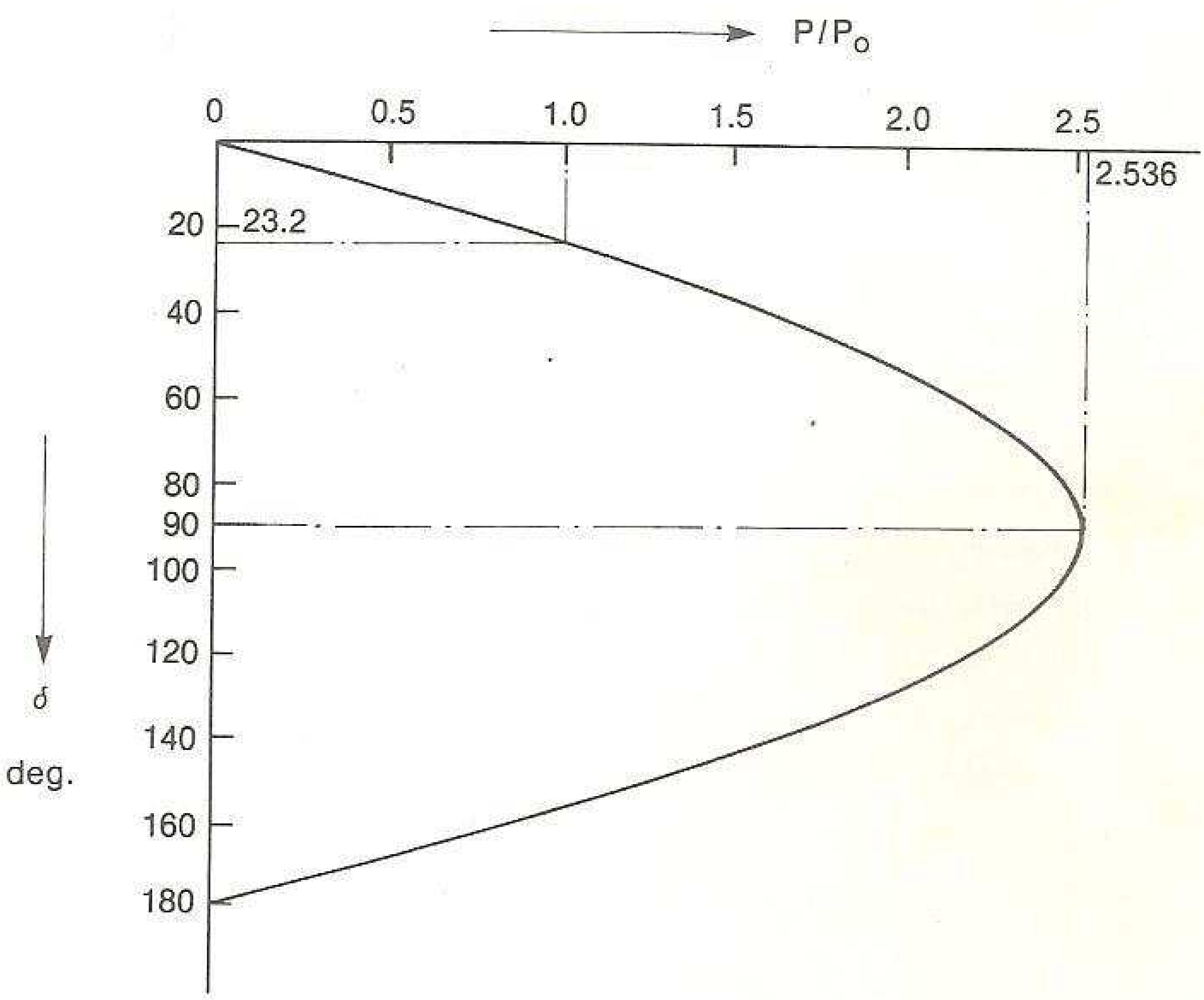
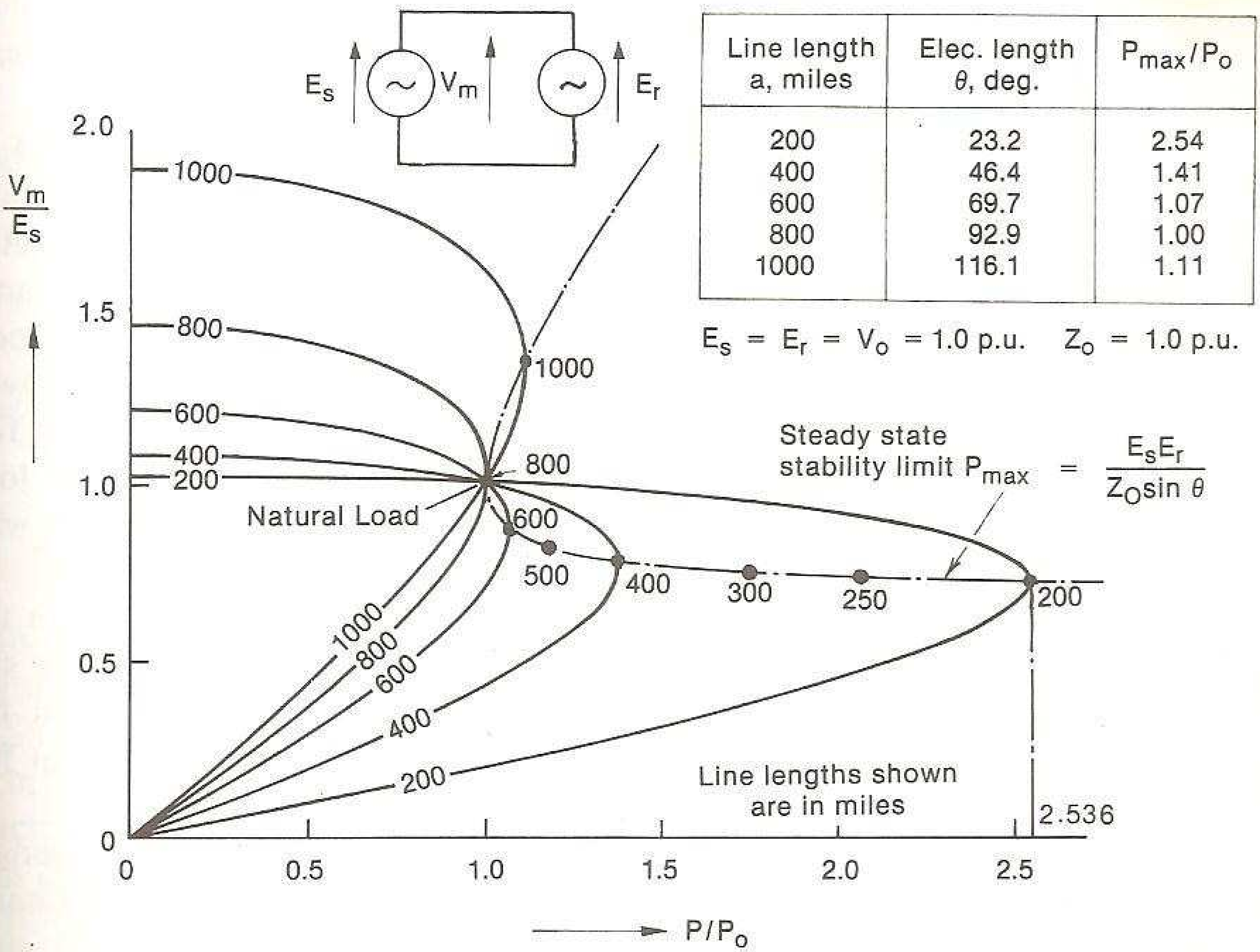


FIGURE 10. Power transmission angle characteristic for 200-mi line.



Line length a, miles	Elec. length θ, deg.	P_{max}/P_o
200	23.2	2.54
400	46.4	1.41
600	69.7	1.07
800	92.9	1.00
1000	116.1	1.11

FIGURE 11. Variation of midpoint voltage with power transmitted along a symmetrical line.

they rotate in synchronism. The terminal voltages are assumed to be held constant by excitation control. The load power can be increased by increasing the torque on the shaft of the motor. This causes the motor to slow down, so that if the generator is assumed to continue at constant speed, the load angle increases. According to Figure 10 and Equation 37, the increased load angle is accompanied by an increase in the transmitted power, which causes the motor to speed up again until a new steady state is attained at the new power level. V_m will now be smaller than it was before, and I_m larger. For successive power increments this process cannot continue indefinitely because there comes a point at which the fractional reduction in V_m exceeds the fractional increase in I_m, and their product $P = V_m I_m$ decreases with any further increase in the transmission angle, however small. This point occurs when $\delta = 90°$, and if $E_s = E_r = V_0$ then $V_m = V_0/\sqrt{2}$. If the load torque is increased slightly, δ increases but the transmitted power now decreases, so that if the generator maintains constant speed the motor will slow down still more and will lose synchronism. The system is unstable, and if this condition is arrived at by the gradual process just described, the power is said to have exceeded the steady-state stability limit $P_0/\sin\theta$. At 200 mi this is $P_0/0.394 = 2.54\ P_0$. The theoretical steady-state stability limit for other line lengths is given in the table in Figure 11 and its locus is plotted also. For lines less than $\lambda/4$ in length (775 mi at 60 Hz), the steady-state stability limit decreases rapidly with increasing line length.

Because of frequent minor disturbances in the power transmitted in any real system, as well as occasional major disturbances caused by faults and switching operations, it is not practical to operate an uncompensated line too near to its steady-state stability limit. A margin is necessary, and, based on experience, a general rule is that the load angle on any uncompensated line should not exceed about 30°, corresponding to a power transmission of half the steady-state limit. If this empirical rule is followed, then the maximum electrical length over which the natural load can be transmitted without compensation is given by Equation 39 with $P = P_0$ and $\delta = 30°$, that is, $\theta = 30°$ or $a = 260$ mi at 60 Hz. A smaller power can be stably transmitted over a longer distance, but in the absence of compensation the maximum permissible line length is still limited by the no-load value of V_m or the reactive power ratings of the synchronous machines (whether absorbing at no load or generating at full load).

It appears from Figure 11 that if the uncompensated line length exceeds one quarter-wavelength, then a flat voltage profile is unattainable at *any* stable level of power transmission. The fact that the steady-state stability limit increases with line length for $a > \lambda/4$ is not of practical interest because of the high voltages, the impractical reactive power requirements, and the high voltage sensitivity associated with the upper parts of the curves for $a > \lambda/4$. Even without these difficulties it would be practically impossible to "maneuver" a line into such an operating condition without passing through an unstable range.

Radial Line with Nonsynchronous Load. Figures 7 and 8 show that there is a maximum power that can be transmitted over a line, even when the load is nonsynchronous. The value of the maximum power can be calculated simply, as follows, for a unity power factor load.

From elementary circuit theory, the maximum power that can be drawn by a unity power factor load from a supply represented as an open-circuit voltage in series with a short-circuit impedance, is given by

$$P_{max} = \frac{E_0 I_s}{2\,(1 + \cos\phi_s)}\,. \tag{40}$$

E_0 is the open-circuit voltage, I_s is the short-circuit current, and ϕ_s is the phase angle between them when the supply is short-circuited. If the supply is regarded as the sending-end emf together with the transmission line up to the receiving-end terminals, then from Equations 2a and b

$$\mathbf{E}_0 = \frac{\mathbf{E}_s}{\cos\theta} \tag{41}$$

and

$$\mathbf{I}_s = \frac{\mathbf{E}_s}{(jZ_0 \sin\theta)} \,. \tag{42}$$

Since $\mathbf{E}_0$ leads $\mathbf{I}_s$ by 90°, $\phi_s = 90°$, $\cos\phi_s = 0$, and from Equation 40,

$$P_{max} = \frac{E_s^2}{Z_0 \sin 2\theta} \,. \tag{43}$$

This equation describes a locus on which lie all the maximum-power points in Figure 8*b*.

Since the load is nonsynchronous, the angle δ cannot be interpreted in terms of the relative angular positions of the rotors of an equivalent machine at the receiving-end and the sending-end generator. The question of the maintenance of synchronism is therefore not an issue.

If the system is operating on the upper part of one of the curves in Figure 8*b*, an increase in power transmission can be caused by a reduction in the effective resistance of the load. In practice this might be done by switching on more lighting load. Alternatively, the load torque on induction motors might be increased, causing them to slow down: the increased slip then reduces the effective resistance of the motors. Reduced load resistance draws more current from the supply, and at unity power factor the voltage decreases (Figure 8*b*). Up to the point of maximum power the product of voltage and current increases, and the system is stable. At the point of maximum power, any further reduction of the effective load resistance produces a reduction in transmitted power.

Effect of Generator Reactances. The internal reactances of the synchronous machines at the end(s) of the line add to the series impedance and alter the phase angle between the internal emf's. This is best illustrated by an example.

Figure 12 shows a symmetrical 200-mi line carrying the natural load P_0. The power system at each end of the line is represented by an equivalent synchronous machine with its step-up transformer. The ratings of these machines are each assumed equal to P_0 and their transient reactances are $x'_d = 0.25$ pu.† The transformer reactances on the same

† The transient reactance is used on the assumption that the generators are fitted with fast-acting voltage regulators. See Reference 9.

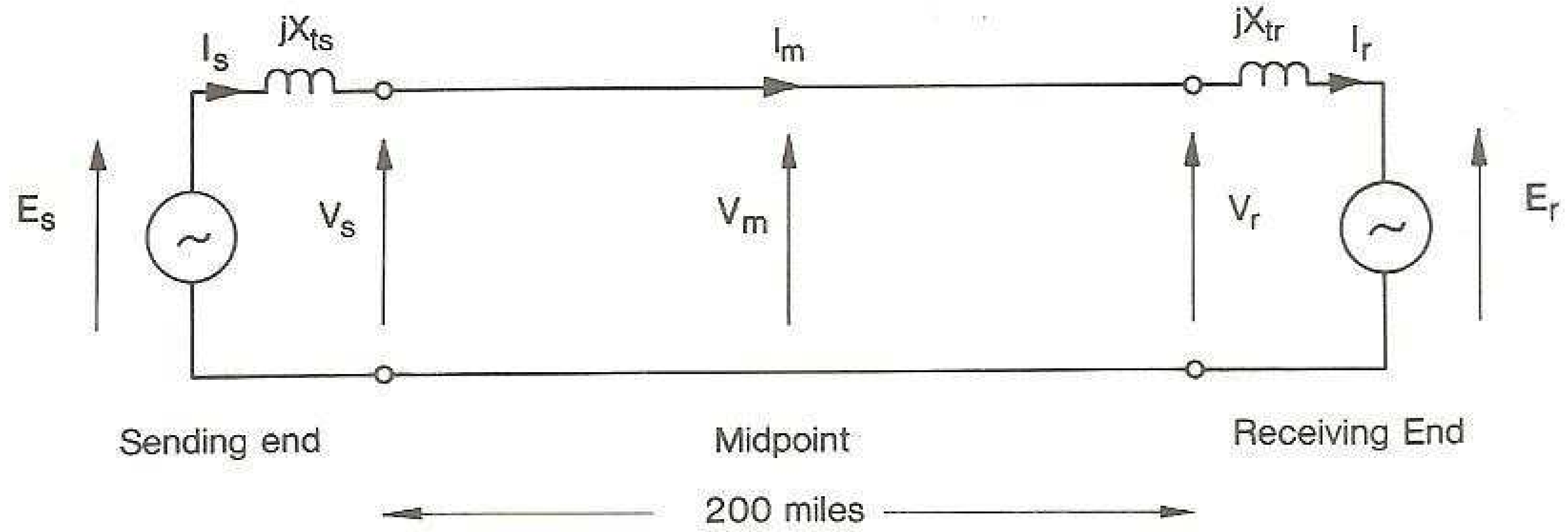


FIGURE 12. Symmetrical line transmitting the natural load P_0: effect of generator reactances at the ends.

per-unit base are assumed to be $x_T = 0.1$ pu so that $x_{ts} = x_{tr} = x'_d + x_T = 0.25 + 0.1 = 0.35$ pu.

The phasor diagram is shown in Figure 13. The excitation of the synchronous machines is adjusted so that $V_s = V_r = V_0 = 1.0$ pu, so that the voltage profile is flat along the line and $V_m = 1.0$ pu. The line angle is 23.2°, as calculated in Section 2.2.3, or from Equation 37. The power factor is unity at both ends, so that if the synchronous machine resistances are neglected the voltage drops across x_{ts} and x_{tr} are in phase quadrature with $\mathbf{I}_s$ and $\mathbf{I}_r$, and both are equal to 0.35 pu, since $I_s = I_r = 1.0$ pu. The total angle δ is 64.2°—nearly 2.8 times that of the line alone.

In the general case of a symmetrical line (with $x_{ts} = x_{tr} = x_t$), the relationship between P and δ (i.e., the phase angle between $\mathbf{E}_s$ and $\mathbf{E}_r$) can be calculated from Equations 25a and b with $P = V_m I_m$ and

$$\mathbf{E}_s = \mathbf{V}_s + jx_{ts}\mathbf{I}_s \,. \tag{44}$$

The result is:

$$P = \frac{E^2}{\left(Z_0 - \dfrac{x_t^2}{Z_0}\right)\sin\theta + 2x_t\cos\theta}\sin\delta\,, \tag{45}$$

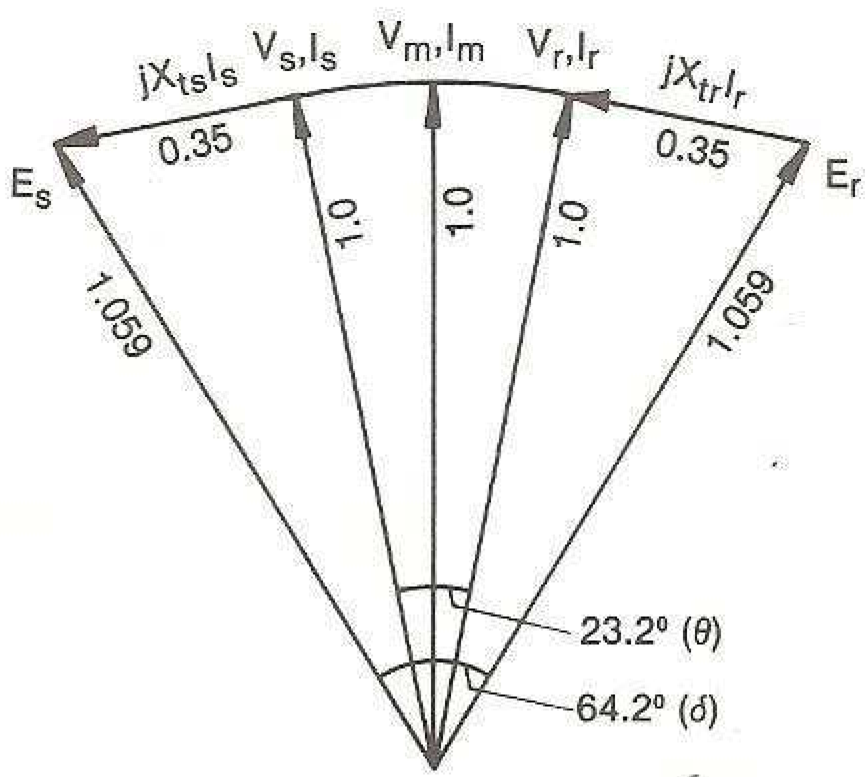


FIGURE 13. Symmetrical line of Figure 12. phasor diagram. (Indicated values are in pu)

where $E_s = E_r = E$. The form of this relationship is exactly as shown in Figure 10, and it reduces to Equation 37 when $x_t = 0$. The maximum power is modified considerably by x_t. The effect depends on the line length and is illustrated in Figure 14, which shows the maximum power as a function of x_t for various line lengths. In practice, uncompensated line lengths do not exceed 100–200 mi, and x_t will generally be less than Z_0. The effect of the generator reactance is therefore to widen the phase angle δ for a given level of power transmission, or to lower the power transmission corresponding to a given angle. (For very long lines there is an upturn in the maximum power when x_t is high. This phenomenon is not of practical interest because operation under such conditions involves very high voltages and reactive-power levels.)

The power transmission relationship can be expressed with E replaced by V_m in Equation 45. The result is

$$P = V_m^2 \frac{\cos\dfrac{\theta}{2} - \dfrac{x_t}{Z_0}\sin\dfrac{\theta}{2}}{Z_0 \sin\dfrac{\theta}{2} + x_t \cos\dfrac{\theta}{2}} \tan\frac{\delta}{2}\,. \qquad (46)$$

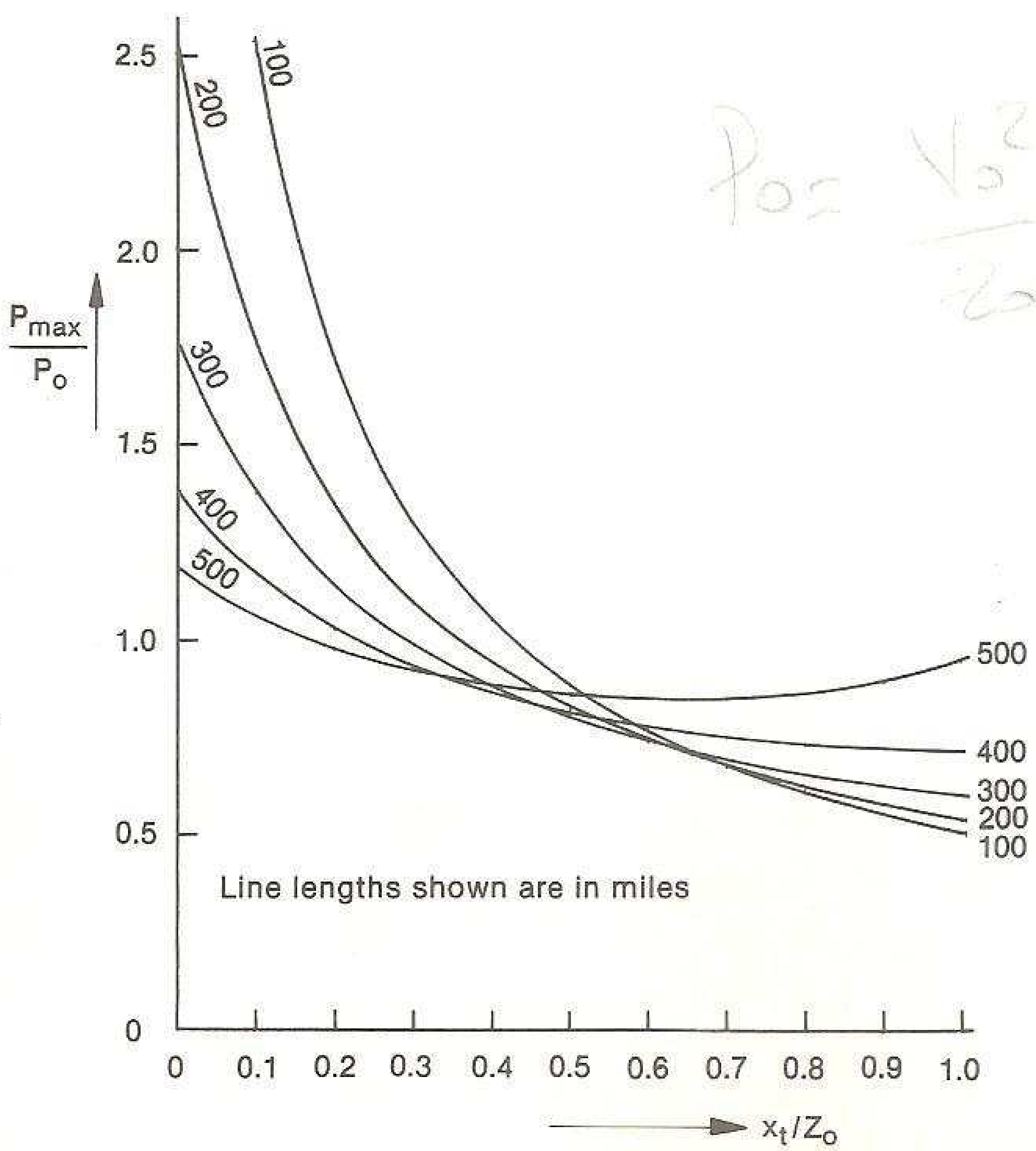


FIGURE 14. Effect of generator reactances on maximum transmissible power for different line lengths.

If $x_t = 0$, this reduces to

$$P = \frac{V_m^2}{Z_0 \tan \frac{\theta}{2}} \tan \frac{\delta}{2} . \tag{47}$$

Alternative Expression for Reactive Power Requirements. In Section 2.2.5, Equation 29 for the reactive power required at the ends of the line assumes that the line is symmetrical and that V_m is known. A useful alternative formula is obtained from Equation 35 for the receiving-end reactive power:

$$Q_r = \frac{V_r(V_s \cos \delta - V_r \cos \theta)}{Z_0 \sin \theta} . \tag{48}$$

A similar procedure can be used to derive the following formula for the sending-end reactive power:

$$Q_s = -\frac{V_s(V_r \cos \delta - V_s \cos \theta)}{Z_0 \sin \theta} . \tag{49}$$

These expressions are valid when the line is not symmetrical, i.e., $V_s \neq V_r$. If $V_s = V_r$ the line is symmetrical and

$$Q_s = -\frac{V_s^2 (\cos \delta - \cos \theta)}{Z_0 \sin \theta} = -Q_r . \tag{50}$$

If $P < P_0$ and $V_s = 1.0$ pu, δ is less than θ, $\cos \delta > \cos \theta$, and Q_s is negative while Q_r is positive. This implies that reactive power is being absorbed at both ends of the line. If $P > P_0$, reactive power is generated at both ends; whereas if $P = P_0$, $Q_s = Q_r = 0$. If $P = 0$, $\cos \delta = 1$ and Equation 50 reduces to Equation 30. The terminal reactive power requirements represented by Equation 50 are illustrated in Figure 15.

For an electrically short line, $\cos \theta \rightarrow 1$ and $Z_0 \sin \theta \rightarrow X_l$, the series reactance of the line, so that Equation 50 reduces to

$$Q_s = \frac{V_s^2 (1 - \cos \delta)}{X_l} = -Q_r . \tag{51}$$

with $V_s = V_r$.

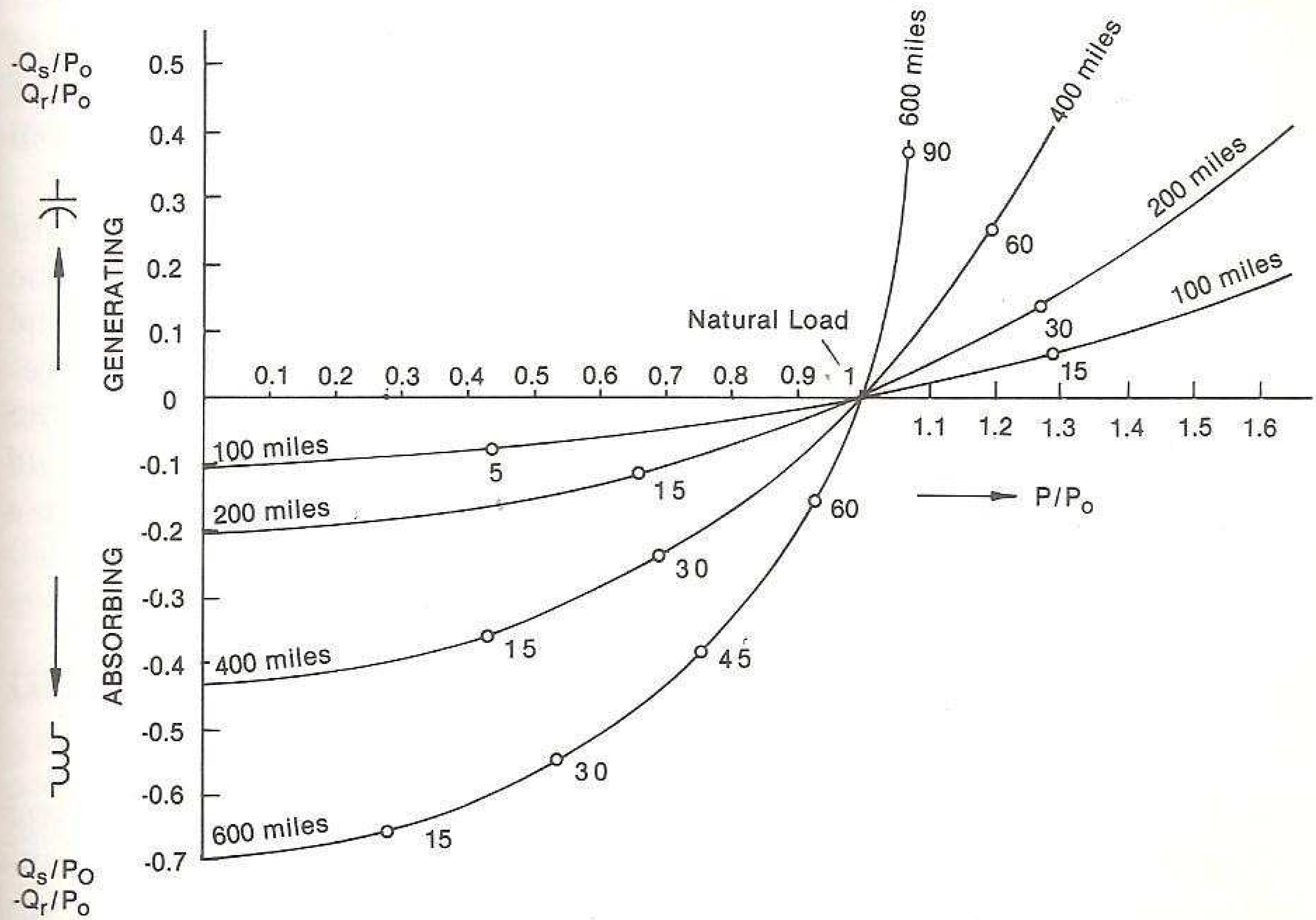


FIGURE 15. Terminal reactive power requirements of symmetrical line, as a function of power transmitted and line length. (Figures show transmission angle δ°)

2.3. COMPENSATED TRANSMISSION LINES

2.3.1. Types of Compensation: Virtual-Z_0, Virtual-θ, and "Compensation by Sectioning"

In this chapter, *compensation* means the modification of the electrical characteristics of a transmission line in order to increase its power transmission capacity while satisfying the fundamental requirements for transmission stated in Section 2.1. With this general objective, a *compensation system* ideally performs the following functions:

1. It helps produce a substantially flat voltage profile at all levels of power transmission;
2. It improves stability by increasing the maximum transmissible power;
3. It provides an economical means for meeting the reactive power requirements of the transmission system.

A figure of merit used to gauge the effectiveness of a compensated system is the product of line length and maximum transmissible power. In

Section 2.2 it was shown, for instance, that at 60 Hz without compensation it is not practical to transmit even the natural load over a distance greater than about 200 mi. Compensated lines enable the transmission of the natural load over greater distances, and shorter compensated lines can carry loads greater than the natural load.

A *flat voltage profile* can be achieved if the effective surge impedance of the line is modified so as to have a *virtual* value, Z_0', for which the corresponding *virtual natural load* V_0^2/Z_0' is equal to the *actual* load. The uncompensated surge impedance is $Z_0=\sqrt{l/c}$. At fundamental frequency this can be written $\sqrt{x_l x_c}$, implying that if the series and/or the shunt reactances x_l and/or x_c are modified (for example by appropriate connection of capacitors or reactors), then the line can be made to have a virtual surge impedance Z_0' and a virtual natural load P_0' for which

$$P_0' = \frac{V_0^2}{Z_0'} = P\,, \qquad (52)$$

where P is the actual power to be transmitted and V_0 is the rated voltage of the line. Since the load P varies, sometimes suddenly, the ideal compensation would be capable of variation also—and without delay. Compensation which can be said to have the objective of modifying Z_0 (or P_0) will be termed *surge-impedance compensation* or Z_0-*compensation.*

Controlling the virtual surge impedance to match a given load (Equation 52) is not sufficient by itself to ensure the *stability* of transmission over longer distances. This is made clear by Figure 11, which shows that without compensation, even under *ideal* conditions (i.e., with no disturbances) the natural load cannot be transmitted stably over distances greater than $\lambda/4(= 775$ mi at 60 Hz$)$. In practice stability is a limiting factor at distances much shorter than this, in the absence of compensation.

Both of the fundamental line parameters Z_0 and θ influence stability through their influence on the transmission angle δ. (See Equation 37). Once a line is compensated in such a way as to satisfy Equation 52—to achieve a flat voltage profile—Z_0' is determined, and the only way to improve stability is to reduce the effective value of θ. Two alternative compensation strategies have been developed to achieve this. One is to apply *series capacitors* to reduce X_l and thereby reduce θ, since $\theta = \beta a = \sqrt{X_l/X_c}$ at fundamental frequency. This strategy might be called *line-length compensation* or θ-*compensation*. The other approach is to divide the line into shorter sections which are more or less independent of one another (except that they all transmit the same power). This might be called *compensation by sectioning.* It is achieved by connecting constant-voltage compensators at intervals along the line. The maximum transmissible power is that of the weakest section, but since this is necessarily shorter than the whole line, an increase in maximum power and, therefore, in stability can be expected.

All three types of compensation may be used together in a single transmission line.

2.3.2. Passive and Active Compensators

It is helpful to distinguish between *passive* and *active* compensators. *Passive* compensators include shunt reactors and capacitors and series capacitors. These devices may be either permanently connected, or switched; but in their usual forms they are incapable of continuous (i.e., stepless) variation. They operate by modifying the natural inductance and capacitance and their operation is essentially static. Apart from switching, they are uncontrolled.

Passive compensators are used only for surge-impedance compensation and line-length compensation. For example, shunt reactors are used to compensate for the effects of distributed line capacitance, particularly in order to limit voltage rise on open circuit or at light load. They tend to increase the virtual surge impedance and reduce the virtual natural load P_0'. Shunt capacitors may be used to augment the natural capacitance of the line under heavy loading. They generate reactive power which tends to boost the voltage. They tend to reduce the virtual surge impedance and to increase P_0'. Series capacitors are used for line-length compensation. Usually a measure of surge-impedance compensation is necessary in conjunction with series capacitors, and this may be provided by an active compensator.

Active compensators are usually shunt-connected devices which have the property of tending to maintain a substantially constant voltage at their terminals. They do this by generating or absorbing precisely the required amount of corrective reactive power in response to any small variation of voltage at their point of connection. They are usually capable of continuous (i.e., stepless) variation and rapid response. Control may be inherent, as in the saturated-reactor compensator; or by means of a control system, as in the synchronous condenser and thyristor-controlled compensators.

Active compensators may be applied either for surge-impedance compensation or for compensation by sectioning. In Z_0-compensation they are capable of all the functions performed by fixed shunt reactors and capacitors and have the additional advantages of continuous variability with rapid response. Compensation by sectioning is fundamentally different in that it is possible *only* with active compensators, which must be capable of virtually immediate response to the smallest variation in power transmission or voltage, that is, their operation is essentially dynamic (in the control engineer's sense). All active compensators except the saturated-reactor type are also capable of acting as passive compensators. Table 4

TABLE 4
Classification of Compensators by Function and Type

Function	Passive	Active
Surge-impedance compensation (Virtual-Z_0 compensation) voltage control, reactive power management	Shunt reactors (linear or nonlinear) Shunt capacitors	Synchronous machines Synchronous condensers Saturated-reactor compensators Thyristor-switched capacitors Thyristor-controlled reactors
Line-length compensation (Virtual-θ compensation) voltage control, reactive power management, stability improvement	Series capacitors	—
Compensation by sectioning Dynamic shunt compensation, stability improvement on longer lines	—	Synchronous condensers Saturated-reactor compensators Thyristor-switched capacitors Thyristor-controlled reactors

summarizes the classification of the main types of compensator according to their usual functions. (Most of these types of compensator had been proposed by the mid-1920's, and several of them can be found in the papers of E.F.W. Alexanderson, who in 1925 discussed the use of shunt reactors controlled by thyratrons, foreshadowing the modern thyristor-controlled reactor compensator.†)

Rapid-response excitation systems used on synchronous machines also have a strong and important compensating effect in a power system. Fitted to the generators at either end of a line, they modify the effective series reactance of the transmission line as a whole, and contribute improvements in both voltage control and stability. They have the effect of reducing the synchronous machine effective reactance to the *transient* reactance x'_d.[9]

The application of reactive power compensation must be done as economically as possible. In some cases the management of reactive

† E.L. Owen, private communication.

power in a power system can be enhanced by modifications to the design of existing (or planned) plant; sometimes this is a cheaper way of improving performance than installing compensation equipment. For example, feedback signals can be used in the automatic voltage regulators of synchronous machines to enhance the stability and enable an increase in power transmission. As another example, shunt reactors and capacitors can often be advantageously relocated after a period of evolutionary change in the loading pattern of the system. More usually, however, compensating equipment is introduced precisely because it is the least expensive way of satisfying the reactive power requirements. This is typically the case when the alternatives are an increase in the number of transmission lines, in the ratings of planned synchronous generators, or in the system voltage.

Other applications and functions of compensators on transmission systems include the management of reactive power flows in order to minimize losses; the damping of power oscillations; the provision of reactive power at dc converter terminals. These are not considered in this chapter.

Both passive and active compensators are in use today and so are all the compensation strategies; virtual-Z_0, virtual-θ, and compensation by sectioning. Although most of the fundamental concepts have a long pedigree, modern activity is considerable. In equipment development, activity is concentrated on the static reactive power controller† or static compensator, to improve its efficiency, reliability, and response characteristics. In the analytical field, attention is focussed on the optimal deployment of compensators, the relative merits of series and shunt compensation schemes for long lines, and the modeling of compensators in power systems on the digital computer.

The remaining sections of this chapter attempt to develop the theory of compensation to the point where the state of the art can be understood in all the main compensation strategies and applications. For further reading the references listed at the end of the chapter should prove helpful.

2.3.3. Uniformly Distributed Fixed Compensation

Modified Line Parameters: Virtual Z_0, θ, and P_0. Compensators are normally connected at the ends of a line or at discrete points along it. In spite of their lumped or concentrated nature, it is useful to derive certain basic relationships for the ideal case of *uniformly distributed* compensation because these relationships are simple and independent of the characteristics of any particular type of compensator. They also give considerable

† This includes the thyristor-controlled reactor (TCR), the thyristor-controlled leakage transformer (TCT), the thyristor-switched capacitor (TSC), and hybrid forms.

physical insight, and help to determine the fundamental nature of the type of compensation required, without reference to extensive computer studies. The formulas derived are in most cases approximately true for practical systems with concentrated compensation because the spacing between compensators is limited by the same factors that limit the maximum length of uncompensated line.

The surge impedance Z_0 of an uncompensated line can be written

$$Z_0 = \sqrt{\frac{l}{c}} = \sqrt{\frac{j\omega l}{j\omega c}} = \sqrt{x_l x_c}\,. \qquad (53)$$

If a uniformly distributed shunt compensating inductance $l_{\gamma sh}$ (H/mile) is introduced, the effective value of the shunt capacitive admittance per mile becomes

$$(j\omega c)' = j\omega c + \frac{1}{j\omega l_{\gamma sh}}$$

$$= j\omega c\,(1 - k_{sh}), \qquad (54)$$

where k_{sh} is the *degree of shunt compensation:*

$$k_{sh} = \frac{1}{\omega^2 l_{\gamma sh} c} = \frac{x_c}{x_{\gamma sh}} = \frac{b_{\gamma sh}}{b_c}\,. \qquad (55)$$

Here $x_{\gamma sh}$ and $b_{\gamma sh}$ are the reactance and susceptance per mile of the shunt compensating inductance. Substituting for $(j\omega c)'$ in Equation 53, the surge impedance has the effective or virtual value:

Compensación reactancia inductiva

$$Z_0' = \frac{Z_0}{\sqrt{1 - k_{sh}}}\,. \qquad (56)$$

If shunt capacitance $c_{\gamma sh}$ is added instead of shunt inductance, then k_{sh} is negative and has the value

$$k_{sh} = -\frac{c_{\gamma sh}}{c} = -\frac{x_c}{x_{\gamma sh}} = -\frac{b_{\gamma sh}}{b_c}\,, \qquad (57)$$

where $x_{\gamma sh}$ and $b_{\gamma sh}$ are the reactance and susceptance per mile of the shunt compensating capacitance.

Shunt inductive compensation, therefore, increases the virtual surge impedance, whereas shunt capacitive compensation reduces it.

In a similar way the effect of uniformly distributed series capacitance $c_{\gamma se}$ on l can be shown to give:

$$Z_0' = Z_0\sqrt{1 - k_{se}}\,, \qquad (58)$$

Compensación reactancia capacitiva

where k_{se} is the *degree of series compensation, given by*

$$k_{se} = \frac{1}{\omega^2 l c_{\gamma se}} = \frac{x_{\gamma se}}{x_l} = \frac{b_l}{b_{\gamma se}}. \tag{59}$$

$x_{\gamma se}$ and $b_{\gamma se}$ are the reactance and susceptance per mile of the series compensating capacitance. The parameters k_{sh} and k_{se} are a useful measure of the reactive power ratings required of the compensating equipment.

Combining the effects of shunt and series compensation,

$$Z_0' = Z_0 \sqrt{\frac{1 - k_{se}}{1 - k_{sh}}}. \tag{60}$$

Corresponding to the virtual surge impedance Z_0' is a virtual natural load P_0' given by V_0^2/Z_0', so that

$$P_0' = P_0 \sqrt{\frac{1 - k_{sh}}{1 - k_{se}}}. \tag{61}$$

The wavenumber β is also modified and has the virtual value

$$\beta' = \beta \sqrt{(1 - k_{sh})(1 - k_{se})}. \tag{62}$$

The electrical length θ is modified according to this equation also:

$$\theta' = \theta \sqrt{(1 - k_{sh})(1 - k_{se})} \tag{63}$$

where $\theta = a\beta$ and $\theta' = a\beta'$. These relationships are shown graphically in Figures 16, 17, and 18.

All the equations in Section 2.2 are valid for the line with uniformly distributed compensation, if the virtual surge impedance Z_0' and the virtual wavenumber β' (or $\theta' = a\beta'$) are substituted for the uncompensated values. This means, for example, that Figure 11 can be used to determine the midpoint voltage of a compensated line under load; Equation 37 can be used to determine the maximum transmissible power and the load angle; and Equation 29 can be used to determine the reactive power requirements at the ends of the line.

Effect of Distributed Compensation on Voltage Control. For any fixed degree of series compensation, additional capacitive shunt compensation increases θ' and P_0' and decreases Z_0', while inductive shunt compensation has the reverse effect. 100% inductive shunt compensation (i.e., $k_{sh} = 1$) reduces θ' and P_0' to zero and increases Z_0' to infinity: this implies a flat voltage profile at zero load, suggesting the use of shunt reactors to cancel the Ferranti effect. Under heavy loading, a flat voltage

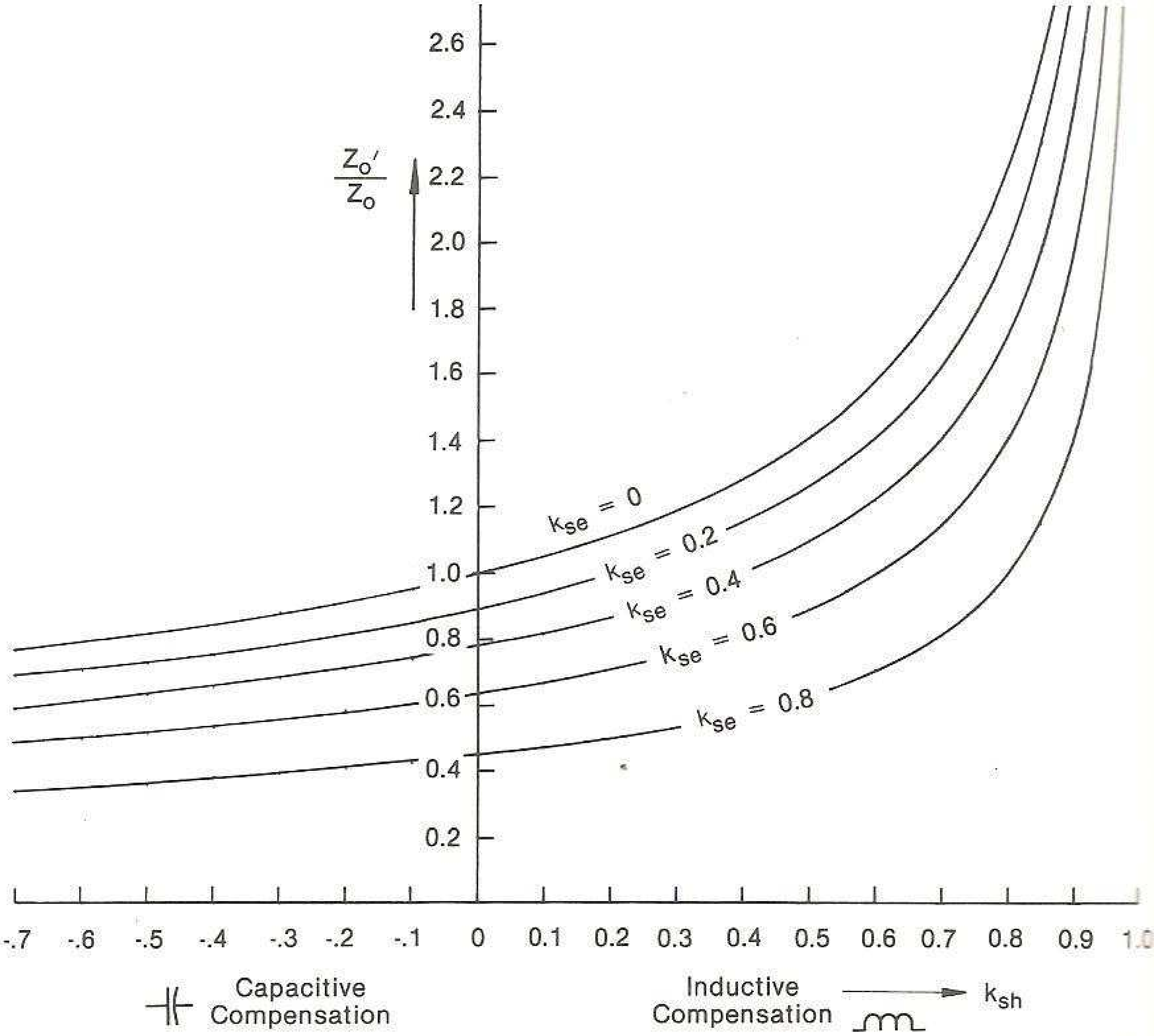


FIGURE 16. Virtual surge impedance Z_0' as a function of k_{sh} and k_{se}. (Uniformly distributed compensation)

profile can be restored by replacing the reactors with shunt capacitors. For example, in order to transmit 1.2 P_0 with a flat voltage profile without series capacitors ($k_{se} = 0$) would require 0.45 pu of distributed shunt capacitive compensation, (from Figure 17); that is, $k_{sh} = -0.45$.

The effect of series capacitive compensation ($k_{se} > 0$) is to decrease Z_0' and θ' and to increase P_0'. Series capacitive compensation can, in principle, be used instead of shunt capacitors to give a flat voltage profile under heavy loading. For example, to transmit 1.2 P_0 with a flat voltage profile without shunt compensation ($k_{sh} = 0$) would require about 0.30 pu of *distributed* series compensation, according to Figure 17; that is, $k_{se} = 0.30$. In reality the lumped nature of series capacitors makes them unsuitable for line voltage control. Their natural application is rather in stabilization, by reducing the virtual line length θ'.

At no-load the midpoint voltage of a compensated symmetrical line is given by Equation 18:

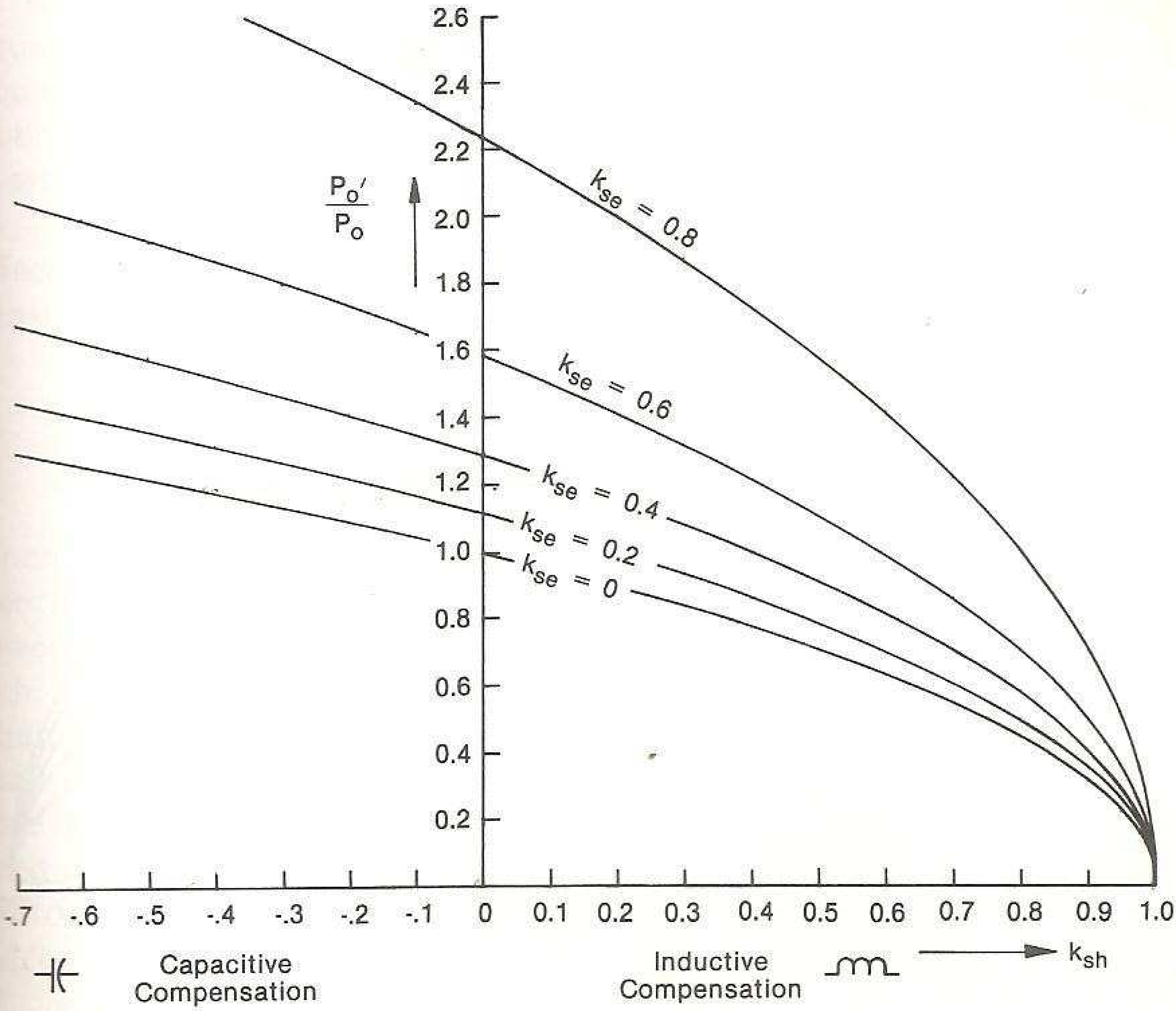


FIGURE 17. Virtual natural load P_0' as a function of k_{se}, k_{sh}.

$$V_m = \frac{E_s}{\cos \frac{\theta'}{2}} \tag{64}$$

It can be seen through their effect on θ' that both series capacitive and shunt inductive distributed compensation tend to reduce the Ferranti voltage rise, whereas shunt capacitive compensation tends to aggravate it.

Effect of Distributed Compensation on Line-Charging Reactive Power. At no-load the line-charging reactive power which has to be absorbed by the terminal synchronous machines is given by

$$Q_s = -P_0' \tan \theta' \tag{65}$$

for a radial line, and

$$Q_s = -P_0' \tan \frac{\theta'}{2} = -Q_r \tag{66}$$

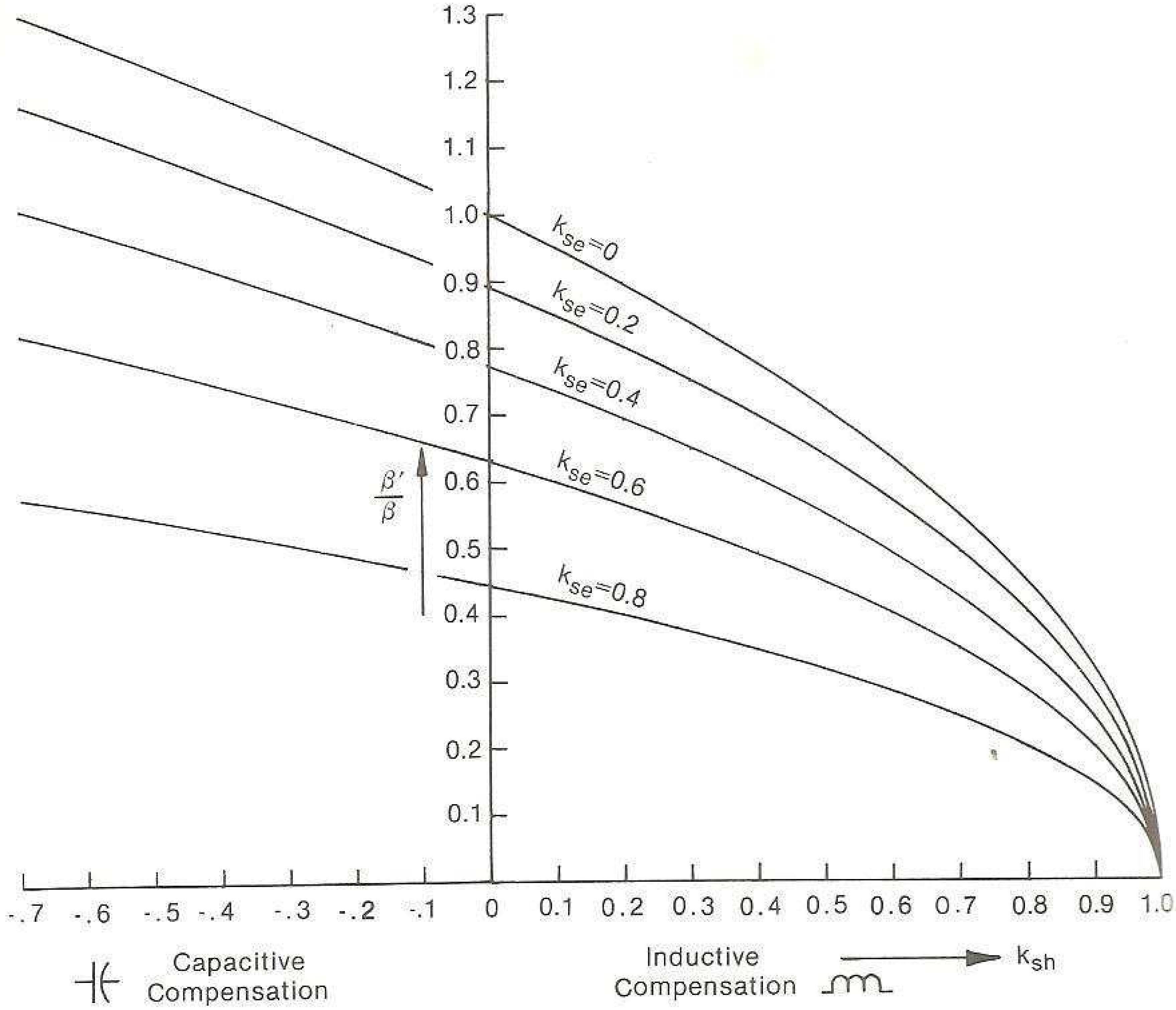


FIGURE 18. Virtual wavenumber β'/β as a function of k_{sh} and k_{se}. (Uniformly distributed compensation)

for a symmetrical line. With a significant degree of either shunt inductive or series capacitive compensation, θ' will tend to be small enough so that $\tan\theta' \simeq \theta'$ and $\tan(\theta'/2) \simeq \theta'/2$. If P_0' and θ are now substituted from Equations 61 and 63 into Equation 66, the factor $\sqrt{1-k_{se}}$ cancels, and for the symmetrical line

$$Q_s = -\frac{P_0\theta}{2}(1-k_{sh}) = -Q_r\,. \tag{67}$$

In the absence of shunt inductive compensation ($k_{sh}=0$) the series-compensated line generates roughly as much line-charging reactive power at no-load as a completely uncompensated line of the same length. If the line is long enough to justify series compensation in the first place, the reactive power absorption required in the terminal synchronous machines at no-load will be excessive. Moreover, the tendency of the synchronous machines to run underexcited at all but the heaviest loads degrades the stability which the capacitors are intended to enhance. This problem can be relieved by means of additional shunt inductive compensation (see Equation 67). Series compensation schemes have virtually always included synchronous condensers and/or shunt reactors for this purpose. Static reactive power controllers can also be used with advantage instead of synchronous condensers.

Effect of Distributed Compensation on Maximum Power. The power transmission Equation 37 can be very approximately written:

$$P \simeq P_0' \frac{\delta}{\theta'} . \tag{68}$$

When $P = P_0'$, $\delta = \theta'$ and the equation is exactly true. The first general objective of a compensation scheme is to produce a high value of P_0', the power level at which the voltage profile is flat. If the system is operated with P near P_0', then δ will necessarily be near θ'. The compensation scheme must now acquire a second objective, which is to ensure that θ' is small enough so that the transmission is stable; that is, that P is not too close to the steady-state stability limit. These objectives will be recognized as the fundamental requirements of transmission (Section 2.1.2).

From Figure 17, a high P_0' can be obtained with series capacitors and/or *capacitive* shunt compensation. On the other hand a low β' (and θ') can be obtained with series capacitors and/or *inductive* shunt compensation, (Figure 18). Only series capacitive compensation contributes to both objectives.

Of course, not all transmission systems requiring compensation require it for *both* objectives. Short lines may require voltage support, equivalent to an increase in P_0', even though their electrical length is much less than 90°. This may be provided by shunt capacitors, provided that θ' does not become excessive as a result. It is common for shunt-compensated lines not exceeding about 200 mi in length to be loaded above the uncompensated natural load. On the other hand, lines longer than about 300–500 mi cannot be loaded even up to the natural load because of the excessive uncompensated electrical length. In these cases the reduction of θ' is the first priority.

The effect of uniformly distributed compensation on the maximum transmissible power (i.e., the steady-state stability limit) is determined from Equation 37 as follows. If the terminal voltages are held constant at V_0, the maximum power is given by

Combining this with Equations 8, 61, and 63,

$$\frac{P'_{max}}{P_0} = \frac{1}{\sqrt{\frac{1-k_{se}}{1-k_{sh}}} \sin\left[\theta\sqrt{(1-k_{se})(1-k_{sh})}\right]} . \tag{70}$$

The form of this equation suggests that a given degree of series compensation has a more pronounced effect on P'_{max} than does the same degree of shunt compensation, because the factors $(1 - k_{sh})$ in the denominator produce opposing influences. This is borne out by numerical examples. The effect of series compensation by itself ($k_{sh} = 0$) is shown in Figure 19 where P'_{max}/P_0 is plotted as a function of k_{se} for various line lengths. The improvement in P'_{max}/P_0 is marked for higher values of $k_{se}(>0.5)$. Very high values of k_{se} can lead to resonance problems, and in practice it is rare to find $k_{se}>0.8$.

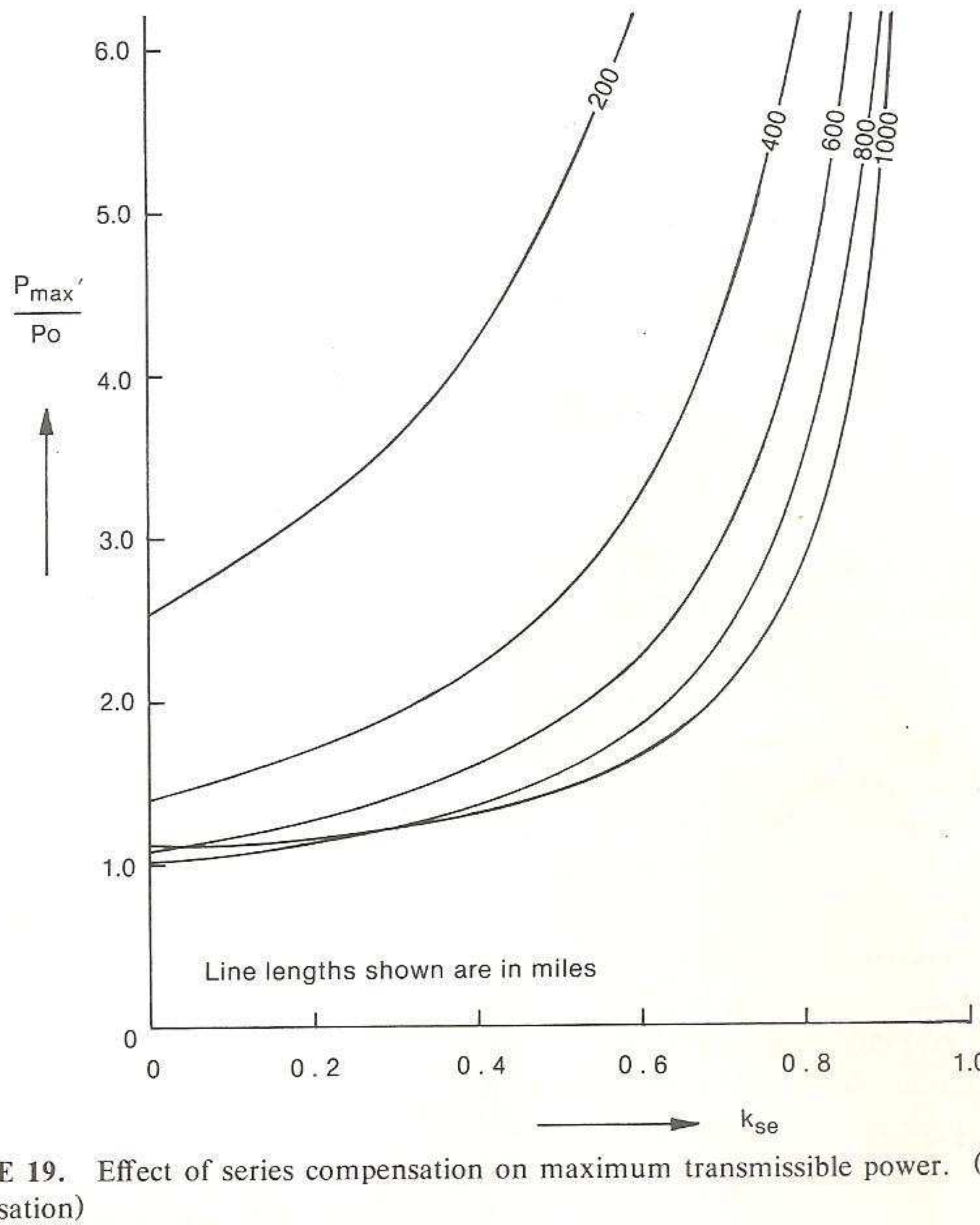


FIGURE 19. Effect of series compensation on maximum transmissible power. (No shunt compensation)

An example of the use of Figures 16 through 19 and their associated equations is as follows. A line 600 miles long has uncompensated values of $\theta = 69.7°$ and $P_{max} = 1.066\ P_0$. The natural load would be unacceptably close to the stability limit. To operate the line with a transmission angle of $\delta = 30°$, the power transfer would have to be no more than about one-half the natural load. With $P = 0.5\ P_0$, Equation 32 shows that in order to maintain a midpoint voltage of 1.0 pu, the terminal voltages have to be $E_s = E_r = 0.869$ pu. These voltages are unacceptably low. Furthermore, from Equation 31 the terminal synchronous machines have to absorb 0.704 kVAr of line-charging reactive power for every kW of power transmitted, operating at a leading (underexcited) power factor of 0.818. With 80% series compensation, Figure 19 gives $P'_{max} = 4.321\ P_0$, Figure 18 gives $\theta' = 0.447\ \theta = 31.2°$, and Figure 17 gives $P'_0 = 2.236\ P_0$. If the power transmitted is now equal to the virtual natural load P'_0, the transmission angle is $\sin^{-1}(P'_0/P'_{max}) = \sin^{-1}(2.236/4.321) = 31.2° = \theta'$ and the voltage profile is flat. The terminal power factors are both unity and no reactive power has to be supplied or absorbed at the ends of the line. This is clearly a marked improvement in the power transmission characteristics of the line.

At no-load the uncompensated midpoint voltage of this line would rise to 1.218 pu (Equation 18). With series compensation the midpoint voltage would rise to only 1.038 pu. The internal reactance of the terminal synchronous machines would tend to cause this voltage to be higher, and it may be desirable to employ shunt reactors to limit it, as well as to relieve the generators of some of the reactive power absorption. Adding uniformly distributed shunt reactance, with $k_{sh} = 0.5$, $\theta' = 22°$, $P'_0 = 1.581\ P_0$, and P'_{max} becomes $4.214\ P_0$. The maximum power, or steady-state stability limit, is not much affected by the additional shunt reactive compensation, but the virtual natural load is appreciably reduced and so is the electrical length. At the virtual natural load the transmission angle is 22.0°, while the midpoint voltage at no-load is 1.019 pu—comparable to that of an uncompensated symmetrical line of length 190 mi.

Ideally it is desirable to compensate a line in such a way that the normal actual load is equal to the virtual natural load P'_0. As already indicated, for stability assessment it is useful to be able to calculate the ratio P'_{max}/P'_0 for the compensated line; from Equations 70 and 61 this is given by

$$\frac{P'_{max}}{P'_0} = \frac{1}{\sin\left[\theta\sqrt{(1-k_{se})(1-k_{sh})}\right]}. \tag{71}$$

The effect of compensation is to improve the ratio P'_{max}/P'_0, that is, to decrease the transmission angle at the virtual natural load. The same result can of course be deduced from Figure 18, since at the virtual natural load $\delta = \theta' = \beta' a$.

A special case arises when $k_{sh} = 1$, that is, with 100% shunt reactive compensation. Equation 70 reduces to

$$\frac{P'_{max}}{P_0} = \frac{1}{\theta(1 - k_{se})}. \tag{72}$$

This relationship is not markedly different from Figure 19. Again, the maximum power is not appreciably affected by shunt compensation. However, with $k_{sh} = 1$, $P'_0 = 0$, that is, a flat voltage profile is obtained only at zero load. The line with 100% shunt compensation behaves exactly as a series inductance or reactance $X_l = \omega a l$. With series (capacitive) compensation the reactance is modified to $X'_l = X_l(1 - k_{se})$.

2.3.4. Uniformly Distributed Regulated Shunt Compensation

This section develops the theory of uniformly distributed *regulated* shunt compensation, which is an ideal system of compensation to which the closest approach in practice is compensation by sectioning, described in detail in Section 2.6.

A line operating at the natural load has a flat voltage profile and, from Equation 39, the transmission angle is then equal to the electrical length of the line, that is, $\delta = \theta$. Considering shunt compensation only ($k_{se} = 0$), suppose that k_{sh} could be continuously *regulated* in such a way that $P'_0 = P$ at all times. (See Equation 61.) Then $\theta' = \delta$ at all times. Therefore,

$$\frac{P}{\delta} = \frac{P'_0}{\theta'}. \tag{73}$$

Substituting for P'_0 and θ' from Equations 61 and 63, respectively,

$$\frac{P}{\delta} = \frac{P_0\sqrt{1 - k_{sh}}}{\theta\sqrt{1 - k_{sh}}} = \frac{P_0}{\theta} = \text{constant}. \tag{74}$$

This implies a *linear* relationship between P and δ, as shown in Figure 20a, suggesting that the maximum transmissible power is *infinite*.

The constant in Equation 74 is the slope of the $P - \delta$ characteristic and is given by

$$\frac{P_0}{\theta} = \frac{V_0^2}{Z_0 \beta a} = \frac{V_0^2}{X_l}. \tag{75}$$

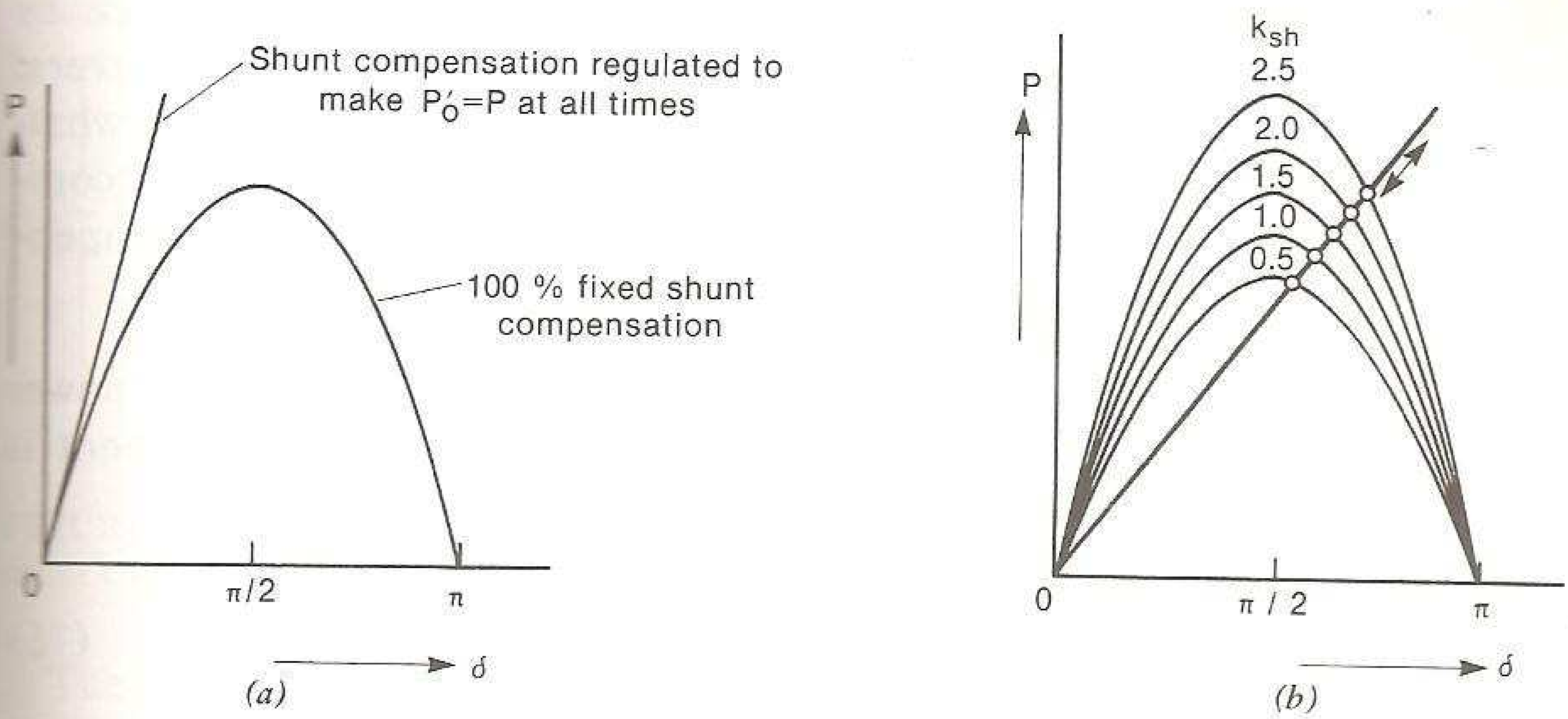


FIGURE 20. (*a*) Effect of regulated shunt compensation on power transmission characteristic. (*b*) Composition of Figure 20*a* from constant-k_{sh} curves.

At zero power the $P-\delta$ line is, therefore, tangent to the sinusoidal $P-\delta$ characteristic of the line with 100% *fixed* shunt compensation ($k_{sh}=1$).

The performance of the *regulated* shunt compensation can be further understood in terms of Figure 20*b*. For every fixed value of k_{sh} there is a sinusoidal $P-\delta$ curve, with P'_{max} given by Equation 69 with $k_{se}=0$. Some of these curves are drawn in Figure 20*b*. As the power varies, the regulating mechanism (whatever it is) adjusts k_{sh} so that the operating point continually shifts from one curve to another in such a way that it always lies on the straight line with positive slope.

It is essential to realize that at large transmission angles the stability is maintained only if k_{sh} can be varied rapidly enough to keep up with any change of P that might occur. If k_{sh} were varied only slowly, a rapid change of P would cause the operating point to move along the sinusoidal $P-\delta$ curve corresponding to the current value of k_{sh} and if $\delta>\pi/2$ rad the system would not be stable. A line with ideal regulated shunt compensation is said to be *dynamically stabilized.*

The regulation of k_{sh} must not only be sufficiently rapid, it must also be *continuous.* If there were any "slack" or dead band in the response of k_{sh} to a change of P, the operating point would move along the "current" sinusoidal $P-\delta$ curve, and when the compensation reacted it would have to over-correct. There would be a tendency towards sustained limit-cycling or hunting of the operating point.

The equations used so far do not suggest how such a compensation scheme could be realized in practice. In practice any compensating devices would not be uniformly distributed, but would be lumped at discrete intervals along the line. The simplest and probably the only practical way to control the compensation is not to try to control k_{sh} directly, but to design the compensation as *constant voltage regulators,* since by continuously forcing the voltage to be constant and equal to V_0 at several points along the line the condition $P_0' = P$ is automatically satisfied whatever the value of P. Such constant-voltage compensators are *active* compensators and can be synchronous condensers, saturated-reactor compensators, or thyristor-controlled compensators.

Reactive Power Required for Compensation. The total reactive power which must be absorbed or supplied by the compensating equipment is easily calculated because the line voltage is constant. It is given by

$$Q_\gamma = -(I^2\omega l - V^2\omega c)a = P_0\theta\left[1 - \left(\frac{P}{P_0}\right)^2\right]. \tag{76}$$

This can also be expressed in terms of the current value of k_{sh}. From Equations 76 and 61 with $k_{se} = 0$ and $P = P_0'$,

$$Q_\gamma = P_0\theta k_{sh}. \tag{77}$$

This equation is valid also for fixed compensation if $P = P_0'$.

The reactive power required is capacitive if $P > P_0'$ and it increases with the square of the transmitted power. There will be a transmitted power above which it is uneconomic to provide the necessary reactive power, and at this level alternatives may need to be considered [e.g., a higher transmission voltage, the use of series as well as shunt compensation, or the use of high voltage direct current (HVDC)]. There are other practical limits to the performance of the regulated shunt compensated scheme. One is the speed of response of the compensators. A second is the problem that if the regulating function fails in one compensator (or if it reaches a reactive-power limit and continues as a fixed shunt susceptance), the stability of the entire system may be seriously impaired. Perhaps the most serious limit is that the dynamic characteristic is straight only while the compensation equipment is within its capacitive current rating. In order to significantly improve the transmission characteristic while maintaining adequate stability margins, a very large amount of compensating capacitance may be required (see Section 2.6).

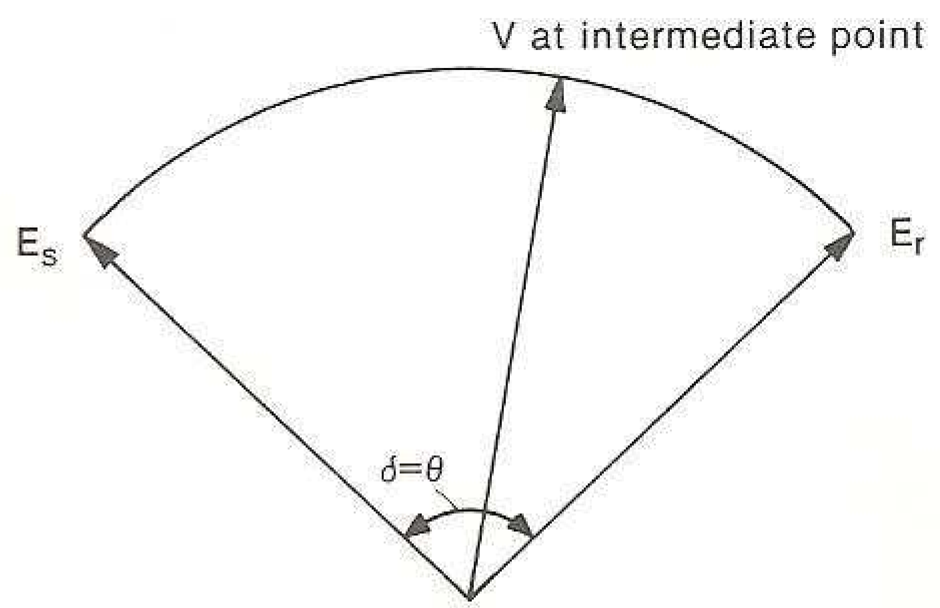


FIGURE 21. Phasor diagram of line with regulated uniformly distributed shunt compensation.

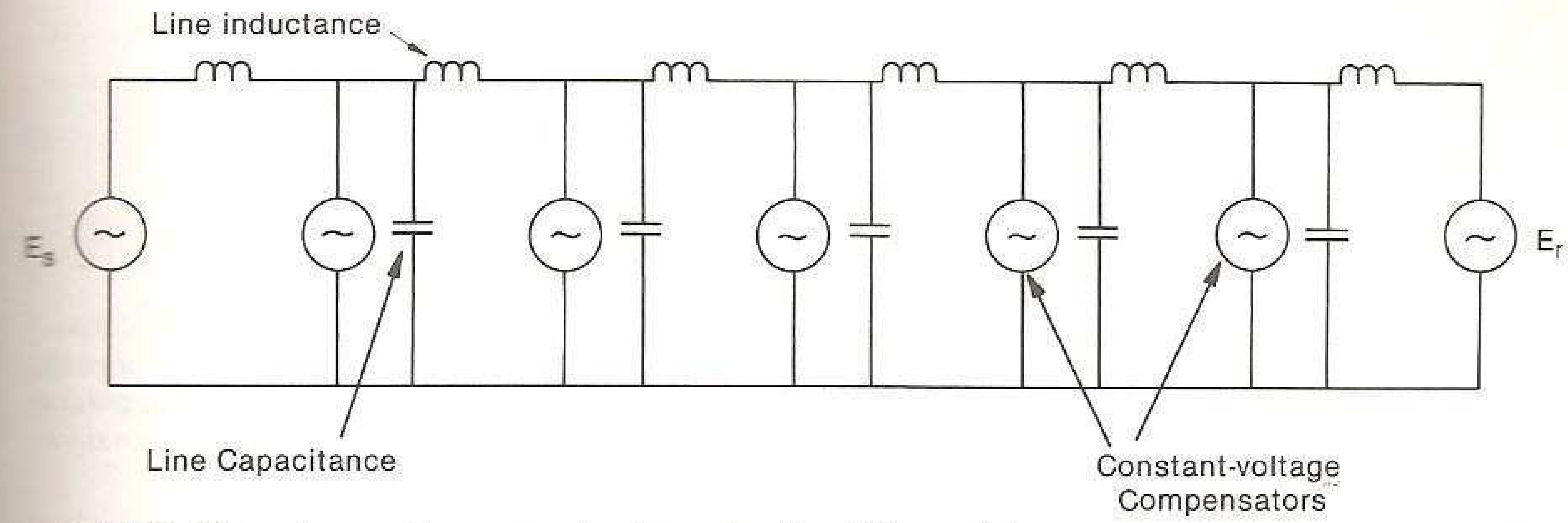


FIGURE 22. Approximate equivalent circuit of line with regulated uniformly distributed shunt compensation.

The phasor diagram of a system with ideal regulated shunt compensation is shown in Figure 21 and Figure 22 is an approximate lumped equivalent circuit. It is of interest to note that the concept of distributed shunt compensation was proposed in 1921 by F. G. Baum[3] as a means of securing constant *voltage* along a long line, but the stability issue was not fully understood at that time and no mention was made of the dynamic nature of this compensation scheme. Baum's compensators would have been synchronous condensers. A truly dynamically compensated line was not built until more than half a century later. One of the largest of these is the 735 kV James Bay Transmission scheme between James Bay and Montréal, Québec, Canada.[16]

2.4. PASSIVE SHUNT COMPENSATION

2.4.1. Control of Open-Circuit Voltage with Shunt Reactors

Shunt compensation with reactors increases the virtual surge impedance (Figure 16) and reduces the virtual natural load, that is, the load at which a flat voltage profile is achieved. With $k_{sh} = 1.0$ the voltage profile is flat at *no-load* (or on open-circuit).

In practice, shunt compensating reactors cannot be uniformly distributed. Instead, they are connected at the ends of the line and at intermediate points—usually at intermediate switching substations. A typical arrangement for a double-circuit line is shown in Figure 23. On a long radial line, the switching stations may occur typically at intervals of between 50 and 250 mi.

In the case of very long lines, at least some of the shunt reactors are permanently connected to the line (as shown in Figure 23) in order to give maximum security against overvoltages in the event of a sudden rejection of load or open-circuiting of the line. On shorter lines, or on sections of line between unswitched reactors, the overvoltage problem is less severe and the reactors may be switched frequently to assist in the hour-by-hour management of reactive power as the load varies. Shunt

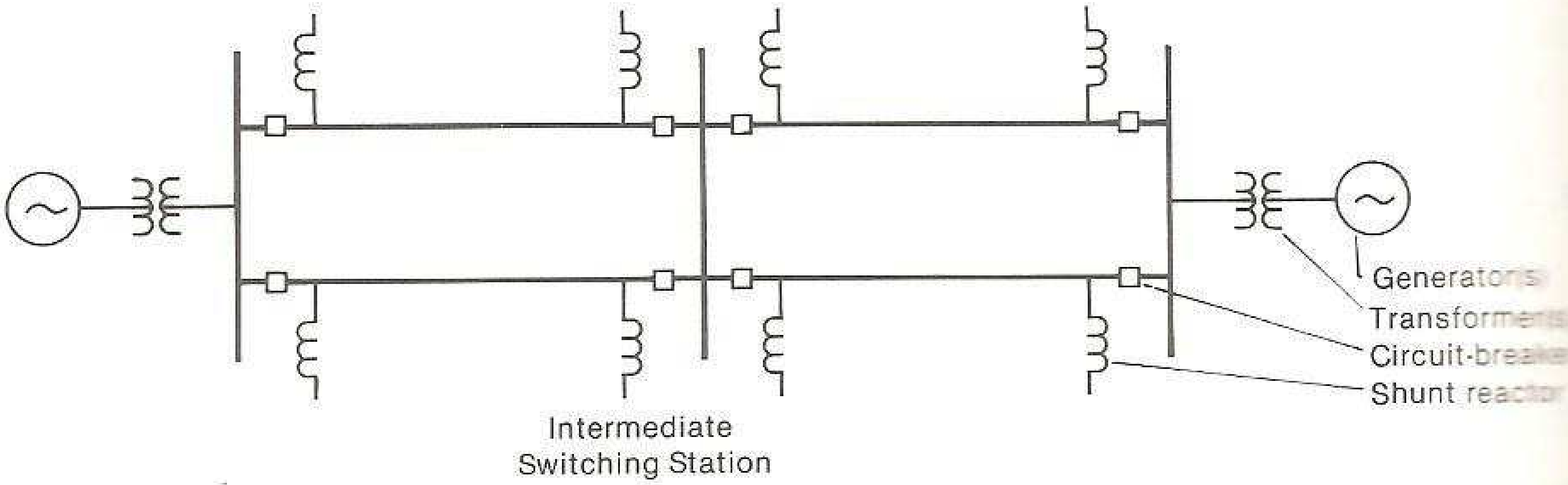


FIGURE 23. Arrangement of shunt reactors on a long-distance high-voltage ac transmission line.

capacitors are usually switched. If there is a sudden load-rejection or open-circuiting of the line, it may be necessary to disconnect them very quickly, to prevent them from increasing the voltage still futher, and also to reduce the likelihood of ferroresonance where transformers remain connected.

Required Reactance Values of Shunt Reactors. The calculation of the optimum ratings and points of connection of shunt reactors and capacitors is generally done by means of extensive load-flow studies, taking into account all possible system configurations.† A simple approach to the problem for a single line is, nevertheless, instructive.

Consider the simple circuit in Figure 24, which has a single shunt reactor of reactance X at the receiving end and a pure voltage source $\mathbf{E}_s$ at the sending end. The receiving-end voltage is given by

$$\mathbf{V}_r = jX\mathbf{I}_r \,. \tag{78}$$

From Equation $2a$,

$$\mathbf{E}_s = \mathbf{V}_r \cos \beta a + jZ_0\mathbf{I}_r \sin \beta a$$

$$= \mathbf{V}_r \left[\cos \theta + \frac{Z_0}{X} \sin \theta\right] . \tag{79}$$

$\mathbf{E}_s$ and $\mathbf{V}_r$ are, therefore, in phase, which is consistent with the fact that no real power is being transmitted. For the receiving-end voltage to be equal to the sending-end voltage, X must be given by

$$X = Z_0 \frac{\sin \theta}{1 - \cos \theta} \,. \tag{80}$$

† A comparative study of several shunt and series compensation schemes was published by Iliceto and Cinieri.[2]

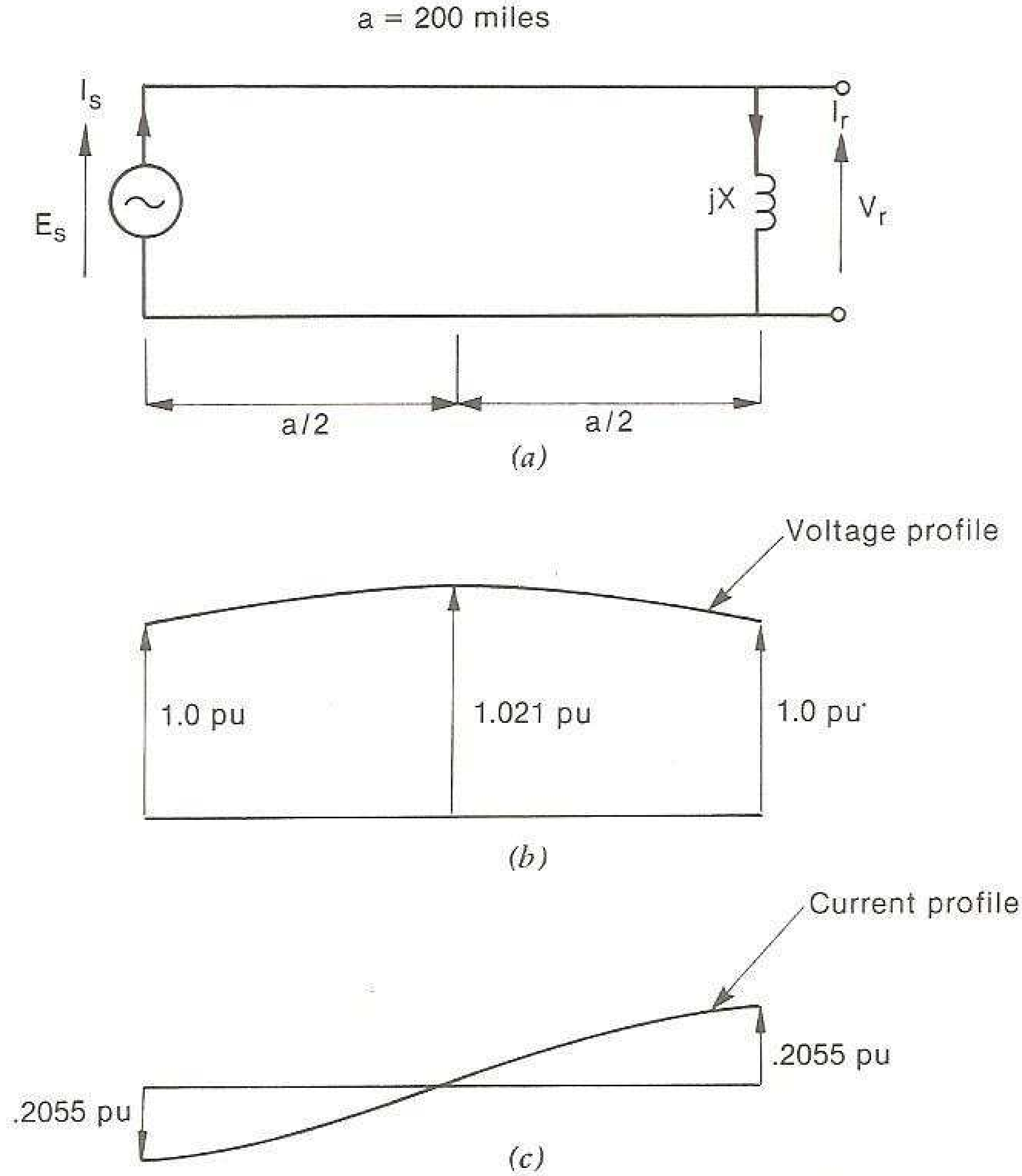


FIGURE 24. Voltage and current profiles of a shunt-compensated line at no-load. (a = 200 mi)

The sending-end current is given by Equation 2*b* as

$$\mathbf{I}_s = j\,\frac{\mathbf{E}_s}{Z_0}\sin\theta + \mathbf{I}_r\cos\theta\,. \tag{81}$$

Making use of Equations 78, 79, and 80, this can be rearranged to give

$$\mathbf{I}_s = j\,\frac{\mathbf{E}_s}{Z_0}\,\frac{1-\cos\theta}{\sin\theta} = j\,\frac{\mathbf{E}_s}{X} = -\mathbf{I}_r \tag{82}$$

since $\mathbf{E}_s = \mathbf{V}_r$. This means that the generator at the sending end behaves exactly like the shunt reactor at the receiving end in that both absorb the same amount of reactive power:

$$Q_s = -Q_r = \frac{E_s^2}{X} = \frac{E_s^2}{Z_0}\left[\frac{1-\cos\theta}{\sin\theta}\right]. \tag{83}$$

The charging current divides equally between the two halves of the line. The voltage profile is symmetrical about the midpoint, and is shown in Figure 24 together with the line-current profile. In the left half of the line the charging current is negative; at the midpoint it is zero; and in the right half it is positive. The maximum voltage occurs at the midpoint and is given by Equation 2a with $x = a/2$:

$$\mathbf{V}_{\mathrm{m}} = \mathbf{V}_{\mathrm{r}}\left[\cos\frac{\theta}{2} + \frac{Z_0}{X}\sin\frac{\theta}{2}\right] = \frac{\mathbf{E}_{\mathrm{s}}}{\cos(\theta/2)}\ . \tag{84}$$

Note that $\mathbf{V}_{\mathrm{m}}$ is in phase with $\mathbf{E}_{\mathrm{s}}$ and $\mathbf{V}_{\mathrm{r}}$, as is the voltage at all points along the line. For a 200-mi line with $E_s = V_0 = 1.0$ pu, the midpoint voltage is 1.021 p.u. and the reactive power absorbed at each end is 0.2055 P_0. These values should be compared with the receiving-end voltage of 1.088 pu and the sending-end reactive-power absorption of $Q_s = 0.429\ P_0$ in the absence of the compensating reactor. For continuous duty at no-load with a line voltage of 500 kV (phase-to-phase) the rating of the shunt reactor shown in Figure 24 would be 68.5 MVAr per phase, Z_0 being 250 Ω.

Equation 84 shows that with the shunt reactor the line behaves at no-load as though it were two separate open-circuited lines placed back-to-back and joined at the midpoint. The open-circuit voltage rise on each half is given by Equation 84.

Multiple Shunt Reactors along a Long Line. The analysis of the simple case shown in Figure 24 can be generalized to describe the behavior of a line divided into n sections by $n - 1$ shunt reactors spaced at equal intervals along the line, together with a shunt reactor at each end. Because of the standing-wave nature of the voltage and current variation along the line, the voltage and current profiles in Figure 24 could be reproduced in every section if every section were of length a and if the *terminal conditions were the same.* The terminal voltages at the ends of the section shown in Figure 24 are equal in magnitude and phase. The currents are equal but opposite in phase. The correct conditions could, therefore, be achieved by connecting shunt reactors of *twice* the susceptance given by Equation 80 at every junction between two sections. The concept of building up a long shunt-compensated line in this way is shown in Figure 25. The shunt reactors at the ends of the line are each of half the susceptance of the intermediate ones. If a is the total length of the composite line, replacing a by a/n in Equation 80 gives the required reactance of each of the intermediate reactors:

$$X = \frac{Z_0}{2}\left[\frac{\sin(\theta/n)}{1 - \cos(\theta/n)}\right]. \tag{85}$$

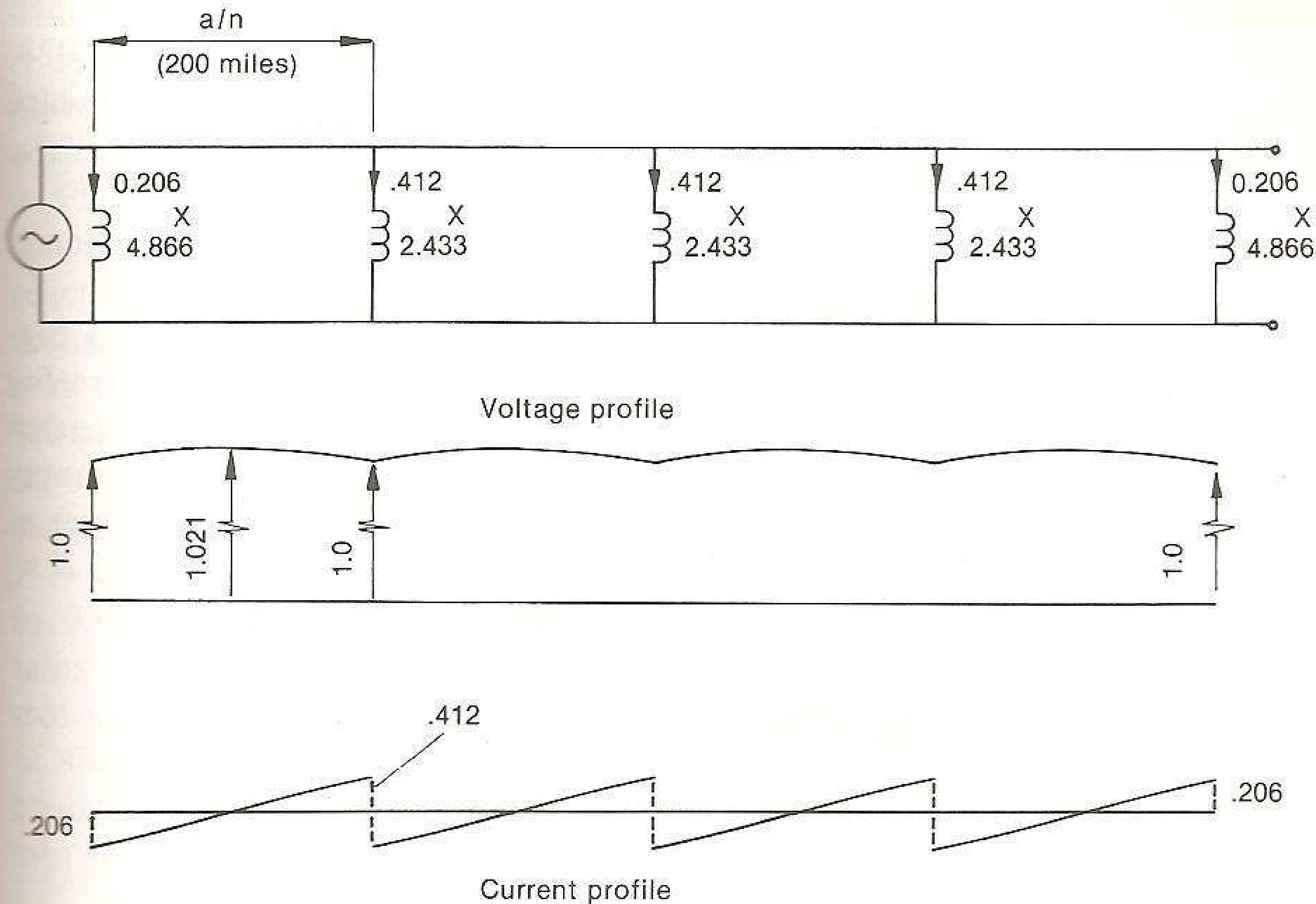


FIGURE 25. Line compensated with multiple shunt reactors at no-load.

In Figure 25, the sending-end generator supplies no reactive current. In practice it would supply, to a first approximation, only the losses.

Each intermediate reactor can be thought of as absorbing the line-charging current or reactive power from two half-sections of line, each of length $a/2n$ on either side of it, whereas the reactors at each end absorb the reactive power from the half-section on one side only. This again explains why the intermediate reactors have twice the susceptance.

In the shunt-compensated systems in Figures 24 and 25, the equivalent degree of compensation k_{sh} is approximately unity. The reactive power absorbed from each half of each 200-mi section is given exactly by Equation 83 as 0.2055 P_0. With *uniformly distributed* compensation and $k_{sh} = 1$, the reactive power to be absorbed from each 100-mi half-section is given by Equation 77 as 0.2027 P_0. The two figures differ by about 1.4% only.

The definition of k_{sh} given in Section 2.3.2 (Equation 55) can be used to define k_{sh} for *lumped* compensation if $b_{\gamma sh}$ is replaced by the total shunt compensating susceptance and b_c by the total shunt capacitance of the line. For example, a typical value of b_c at 500 kV is 8 μS/mi ($c =$ 0.0212 μF/mi) giving a line-charging reactive power of 2 MVAr/mi (three-phase). If $Z_0 = 250\ \Omega$, then from Equation 80 the receiving-end shunt reactor has $X = 1216\ \Omega = 4.866$ pu, $B = 8.22 \times 10^{-4}$ S, and a reactive power of 205.5 MVAr, corresponding to the charging reactive power of half the line, which is 100 × 2 = 200 MVAr. The lumped value of k_{sh} in this example is $8.22 \times 10^{-4}/(100 \times 8 \times 10^{-6}) = 1.028$. The use and interpretation of k_{sh}, therefore, has to be treated with care over longer line sections.

2.4.2. Voltage Control by Means of Switched Shunt Compensation

The voltage regulation diagram for the line of Figure 24 is shown in Figure 26 for three different power factors. The curves labeled "U" are for the uncompensated line; those labeled "L" apply when the shunt reactor discussed in Section 2.4.1 is connected; and those labeled "C" apply when a capacitor of equal reactance is connected instead of the reactor.

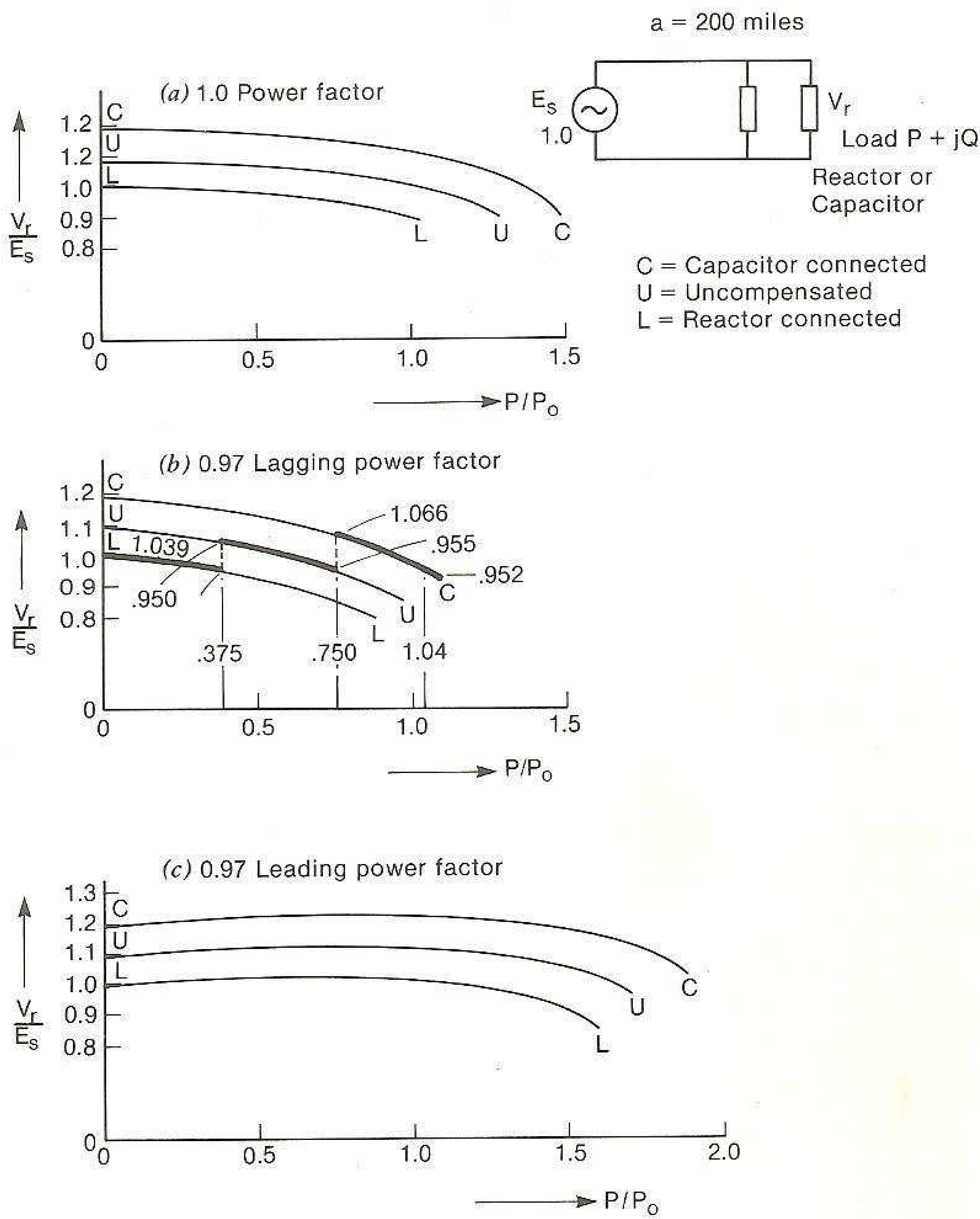


FIGURE 26. Voltage control by means of switched shunt compensation.

The fact that the "L" curve lies below the "U" curve in every case reflects the fact that on the uncompensated line the line-charging reactive power supports the voltage to a significant degree under load. This advantage is lost if the shunt reactor is permanently connected.

Figure 26*b* illustrates the principle by which the receiving-end voltage can be kept more or less constant as the load varies, by switching the reactor and the capacitor. In this example the load power factor is assumed to be 0.97 lagging. At zero load, the shunt reactor is connected and reduces the uncompensated open-circuit voltage from 1.088 pu to 1.0 pu. It remains connected until the power transfer reaches 0.375 P_0, at which level the receiving-end voltage has decreased to 0.95 pu. The reactor is then disconnected and the line remains uncompensated between $P = 0.375\ P_0$ and $P = 0.75\ P_0$, at which level the voltage has decreased to 0.955 pu. When $P = 0.75\ P_0$ the capacitor is connected, and it sustains the voltage above 0.95 pu until P reaches 1.04 P_0. The voltage control in this illustrative example is very coarse. In practice, the switching of shunt reactors and/or capacitors is coordinated with the control of tap-changing transformers and other voltage-regulating equipment to maintain the voltage within narrower limits than are indicated in Figure 26*b*.

2.4.3. The Midpoint Shunt Reactor or Capacitor

The shunt reactor or capacitor located at the midpoint of a symmetrical line is a special case of the line with $n - 1$ reactors considered in Section 2.4.1. It has $n = 2$. It is useful to study this case in detail because it can be meaningfully compared with the series-compensated line treated in Section 2.5 and the line compensated by sectioning in Section 2.6. Its main application is in the control of line voltage and power factor, as will be shown.

Each half of the line is represented by its π-equivalent circuit as in Figure 27*a*. The terminal synchronous machines are assumed to have constant voltage and zero internal reactance. The two capacitive shunt susceptances $B_c/4$ connected at the ends can be omitted from the analysis if it is remembered that the terminal synchronous machines absorb their reactive power at all times. To account for this, $\mathbf{I}_s$ is written $\mathbf{I}_s = \mathbf{I}_s' + j(B_c/4)\mathbf{E}_s$, and similarly for $\mathbf{I}_r$. In effect, the two extreme quarters of the line length have 100% compensation of their shunt capacitance. The central half of the line is compensated by a single shunt reactor or capacitor, which may be switched on or off, but whose susceptance cannot be varied continuously. The degree of compensation for the central half is given by

$$k_m = \frac{B_\gamma}{\frac{1}{2} B_c}. \tag{86}$$

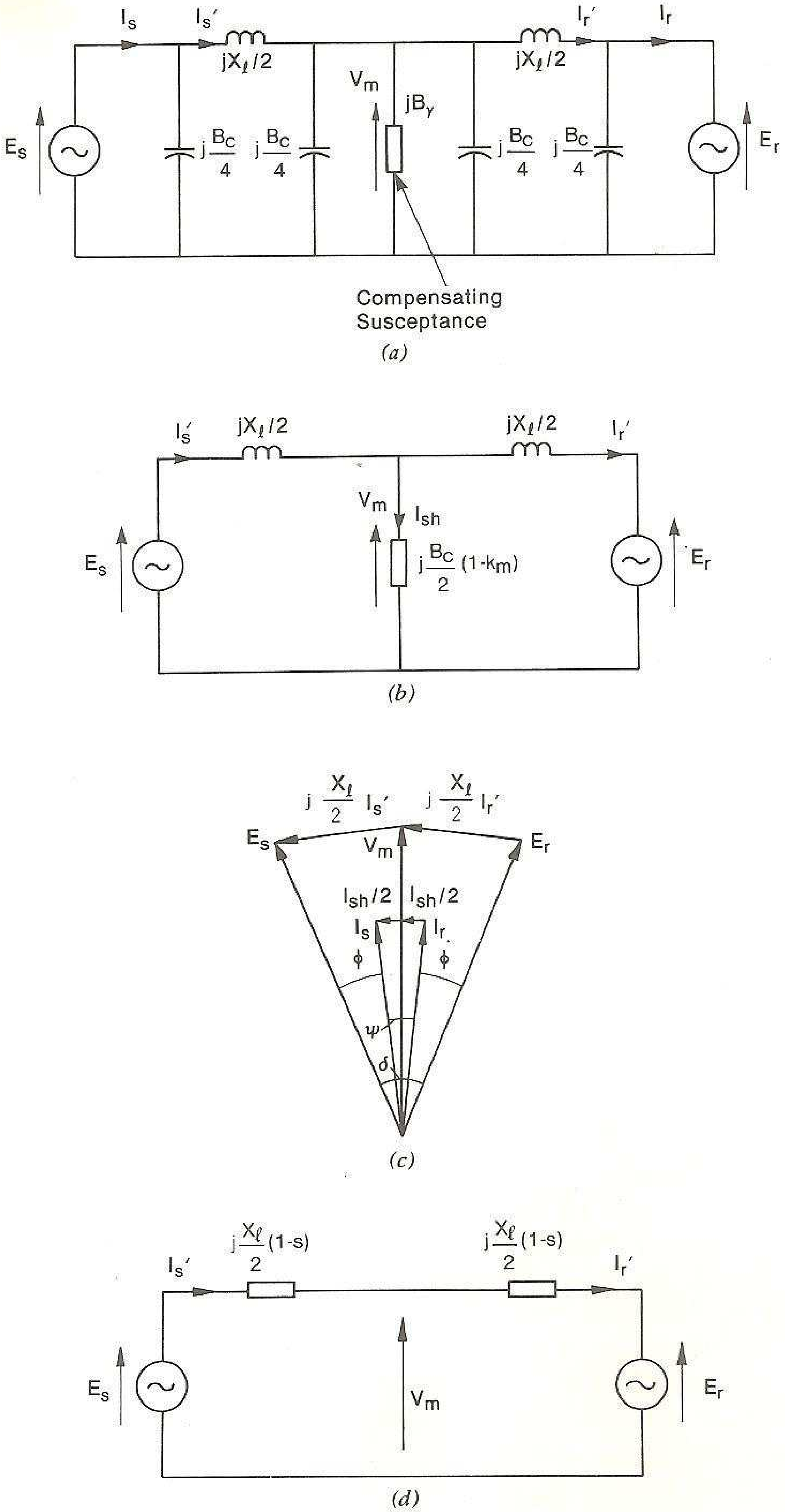


FIGURE 27. Analysis of line compensated with midpoint reactor or capacitor.

k_m is arbitrarily taken as positive if B_γ is inductive, negative if B_γ is capacitive, and is unity when $B_\gamma = B_c/2$, B_c being the shunt susceptance of the whole line.

At fundamental frequency, the equivalent circuit reduces to Figure 27*b*. The phasor diagram is given in Figure 27*c*. The basic relationships are

$$P = V_m I_m \,, \tag{87}$$

$$V_m = \mathbf{E}_r + j\,\frac{X_l}{2}\,\mathbf{I}_r' = \mathbf{E}_s - j\,\frac{X_l}{2}\,\mathbf{I}_s' \,, \tag{88}$$

and

$$\mathbf{I}_{sh} = \mathbf{I}_s' - \mathbf{I}_r' = 2(\mathbf{I}_s' - \mathbf{I}_m) = 2(\mathbf{I}_m - \mathbf{I}_r') = j\,\frac{B_c}{2}\,(1 - k_m)\mathbf{V}_m \,. \tag{89}$$

From these it can be shown that with $E_s = E_r = E$,

$$P = \frac{E^2}{X_l(1 - s)} \sin\delta \,, \tag{90}$$

where

$$s = \frac{X_l}{2}\,\frac{B_c}{4}\,(1 - k_m) \,. \tag{91}$$

The midpoint voltage can be expressed as

$$V_m = \frac{E \cos(\delta/2)}{1 - s} \,, \tag{92}$$

which shows that with $s < 1$ the midpoint compensation increases the midpoint voltage by the factor $1/(1 - s)$. This factor of increase tends to offset the voltage drop in the series reactance of the line and, therefore, mitigate the midpoint voltage sag under load. If $\cos(\delta/2)/(1 - s) > 1$ the midpoint voltage exceeds the terminal voltages even under heavy load.

If $k_m = 1$ the shunt capacitance is canceled along the entire line length, making the virtual natural load zero. The compensating susceptance is then a reactor of admittance $B_c/2$ (Equation 86). Only at no-load is the voltage profile flat. The line is practically reduced to its series impedance. If the terminal voltages are adjusted to keep $V_m = V_0 = 1$ pu while the power transfer is increased, then their values are given by the equation

$$E_s = E_r = V_0 \sqrt{1 + \left(\frac{x_l}{2}\right)^2 \left(\frac{P}{P_0}\right)^2}\,, \tag{93}$$

which can be deduced from the phasor diagram. Note that here we have written $x_l = X_l / Z_0 = \theta = \beta a$.

Among other equations of interest for the midpoint-compensated symmetrical line are the following. The terminal currents are given by

$$\mathbf{I}_s = \mathbf{I}_r^* = I_m \left(1 - \frac{B_c X_l}{8}\right) + \mathrm{j} V_m \frac{B_c}{4}(2 - k_m - s)\,. \tag{94}$$

The terminal reactive powers are given by

$$Q_s = -Q_r = \frac{P^2}{V_m^2}\frac{X_l}{2}\left\{1 - \frac{B_c X_l}{8}\right\} - V_m^2 \frac{B_c}{4}(1 - s)(2 - k_m - s). \tag{95}$$

This expression includes the reactive power of the extreme capacitive branches of the π-equivalent circuits, which the terminal synchronous machines must absorb at all times.

The reactive power of the midpoint compensating susceptance is given by

$$Q_\gamma = E^2 \frac{B_c k_m}{4(1 - s)^2}(1 + \cos\delta)\,. \tag{96}$$

Note that Q_γ is positive for a reactor ($k_m > 0$) and negative for a capacitor ($k_m < 0$).

An example illustrates the features of the midpoint-compensated line. For a 200-mi line $B_c / Y_0 = X_l / Z_0 = \theta = 0.4054$ pu.† For 100% compensation of the line capacitance $B_\gamma = B_c/2 = 0.2027$ per-unit of Y_0. At 500 kV with $Z_0 = 250\Omega$ the required shunt reactor would have a reactance of

$$X_\gamma = \frac{2}{B_c} Z_0 = \frac{2}{0.4054} \times 250 = 1233\Omega \tag{97}$$

† The short-section π-equivalent circuit is used in this example.

(phase-neutral), and its reactive-power rating would be $[500/\sqrt{3}]^2/1233 = 67.6$ MVA per phase. This agrees with the approximate figure obtained by reckoning 2 MVA/mi for the three phases over the central 100-mi section of the line (see Table 3). At no-load the terminal synchronous machines must each absorb ½ × 3 × 67.6 = 101.4 MVA, which is about one-tenth of the natural load. With the reactors connected, $k_m = 1$ and $s = 0$. The voltage profile is approximately flat, with $E_s = E_r = V_m = 500$ kV (phase-to-phase). Without the reactor, the midpoint voltage would be $V_m = 1.021$ pu $= 510.5$ kV (see Figure 5*b*).

With the midpoint reactors connected, at the natural load $P = P_0 = 1000$ MW (three-phase), and from Equation 90, $\delta = 23.92°$. The midpoint voltage is given by Equation 92: with $E = 1.0$ pu $= 500$ kV, $V_m = \cos(\delta/2) = 0.978$ pu $= 489$ kV (phase-to-phase). The voltage profile is as shown in Figure 28*b*. The terminal reactive powers are given by Equation 95, that is, 212 MVAr generation at each end, giving a terminal power factor of 0.978 lagging at the sending end and 0.978 leading at the receiving end.

If the reactor is replaced by a capacitor of equal susceptance, $k_m = -1$ and from Equation 91

$$s = \frac{0.4054\ Z_0}{2} \times \frac{0.4054\ Y_0}{4}\ [1 - (-1)] = 0.0411\,. \tag{98}$$

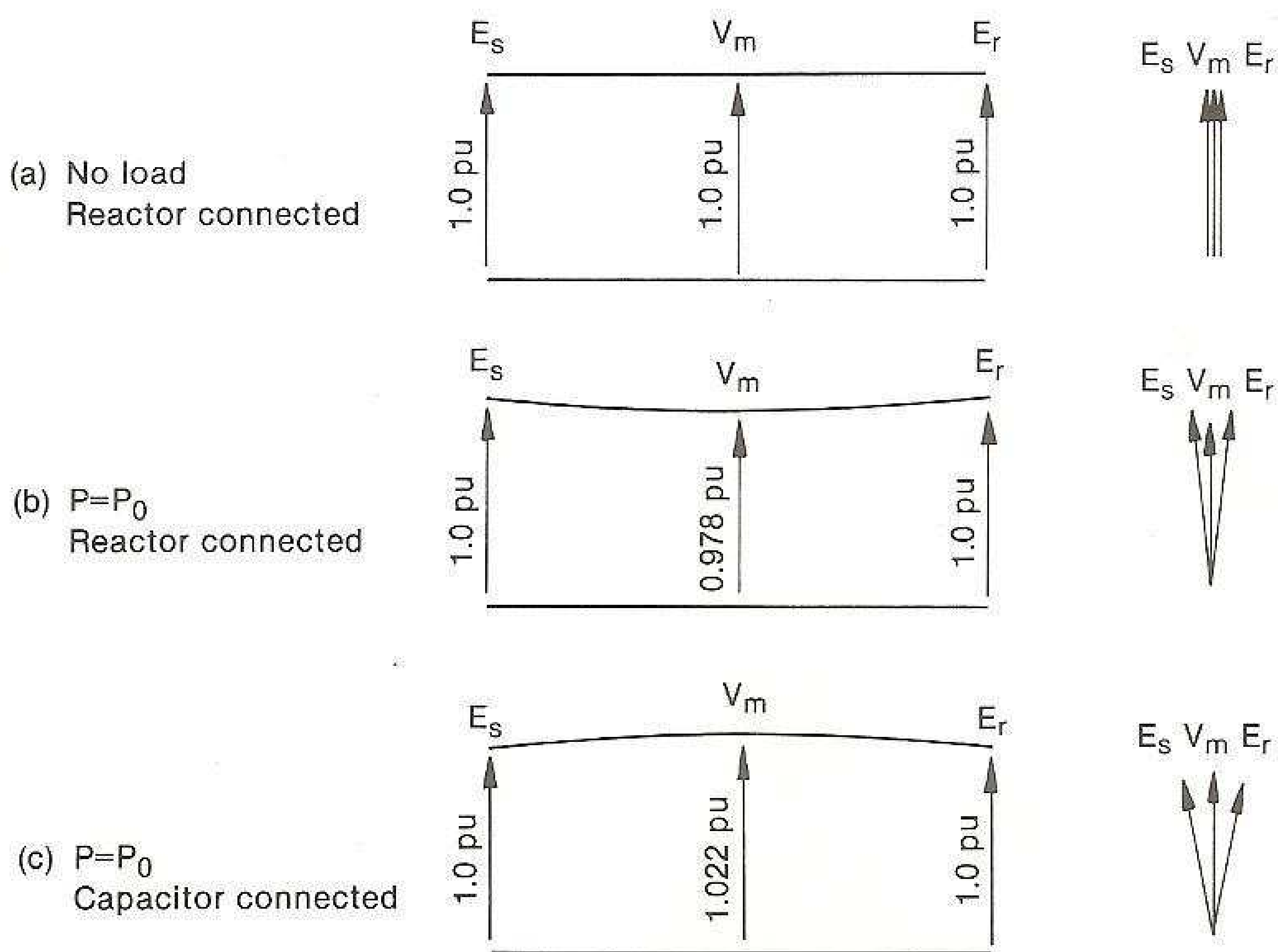


FIGURE 28. Voltage profiles of line with midpoint shunt compensation.

At the natural load, from Equation 90, $\delta = 22.88°$ and from Equation 92, $V_m = 1.022$ pu. The terminal reactive powers are now each 9 MVAr absorption at each end. As would be expected with such a low value of s, the stabilizing influence of the midpoint capacitor is insignificant, although the midpoint *voltage support* is worthwhile.

In the preceding example the midpoint susceptance is much more effective in controlling the line voltage and the terminal power factors than in reducing the transmission angle. It is for voltage and power factor control that this type of compensation is normally used. It appears from Equation 90 that a significant effect on the transmission angle would be achieved only if s were an order of magnitude larger than in the example: this would be the case if the line length were very much longer and/or if the capacitive compensating susceptance were extremely large. However, since the midpoint *voltage* is also subject to the factor $1/(1 - s)$, with so much compensation it would either become unacceptably high, or become excessively sensitive to the switching of the capacitor, or both. For these reasons the single passive shunt susceptance is not a practical way of increasing the maximum transmissible power of a long line. For shunt stabilization of a long line, the susceptance has to be large and *dynamically controlled,* as for example in the thyristor-controlled or saturated-reactor compensators (see Section 2.6).

The form of Equation 90 suggests a parallel between the midpoint-compensated line and the series-compensated line because the coefficient of $\sin \delta$ is of the same form as Equation 109 with k_{se} replaced by s. In this sense s can be interpreted as an equivalent degree of series compensation. The series-capacitor effect in the transmission-angle characteristic is weak and is obtained only with $s < 1$. This requires $k_m < 1$, which says that if B_γ is inductive it must not be so large as to compensate all the shunt capacitance of the central half of the line. In other words, the *shunt capacitance of the line has a stabilizing influence on the power transmission*—which can be enhanced if B_γ is made capacitive ($k_m < 0$), or weakened if B_γ is made inductive ($k_m > 0$).

2.5. SERIES COMPENSATION[11-13]

2.5.1. Objectives and Practical Limitations

As discussed in Section 2.3, the central concept in series compensation is to cancel part of the reactance of the line by means of series capacitors. This increases the maximum power, reduces the transmission angle at a given level of power transfer, and increases the virtual natural load. The line reactance, however, being effectively reduced, now absorbs less of the line-charging reactive power, often necessitating some form of shunt-inductive compensation.

As a means of reducing the "transfer reactance" between the ends of a line, the series capacitor finds two main classes of application:

1. It can be used to increase the power transfer on a line of any length. Sometimes a series capacitor is used to increase the load share on one of two or more parallel lines—especially where a higher-voltage line overlays a lower-voltage line in the same corridor.
2. It can be used to enable power to be transmitted stably over a greater *distance* than is possible without compensation. On long lines shunt inductive compensation is usually also necessary in order to limit the line voltage.

A practical upper limit to the degree of series compensation is of the order of 0.8. With $k_{se} = 1$ the effective line reactance would be zero, so that the smallest disturbance in the relative rotor angles of the terminal synchronous machines would result in the flow of large currents. The circuit would also be series resonant at the fundamental frequency, and it would be difficult to control transient voltages and currents during disturbances.

The capacitor *reactance* is determined by the desired steady-state and transient power transfer characteristics, as well as by the location of the capacitor on the line. Its location is decided partly by economic factors and partly by the severity of fault currents (which are a function of the capacitor location). The voltage rating is determined by the severity of the worst anticipated fault currents through the capacitor and any bypass equipment—"severity" being a function of duration as well as magnitude.

Clearly it is not practicable to distribute the capacitance in small units along the line. In practice lumped capacitors are installed at a small number of locations (typically one or two) along the line. This makes for an uneven voltage profile,[(2)] as will be seen.

2.5.2. Symmetrical Line with Midpoint Series Capacitor and Shunt Reactors

The case studied in this section is a lossless, symmetrical line with a midpoint series capacitor on either side of which are connected two equal shunt reactors (see Figure 29*a*).† The capacitor is shown split into two equal series parts for convenience in analysis. The purpose of the shunt reactors is to control the line voltage; this is illustrated by an example in Section 2.5.3.

† Other configurations are studied in Reference 2.

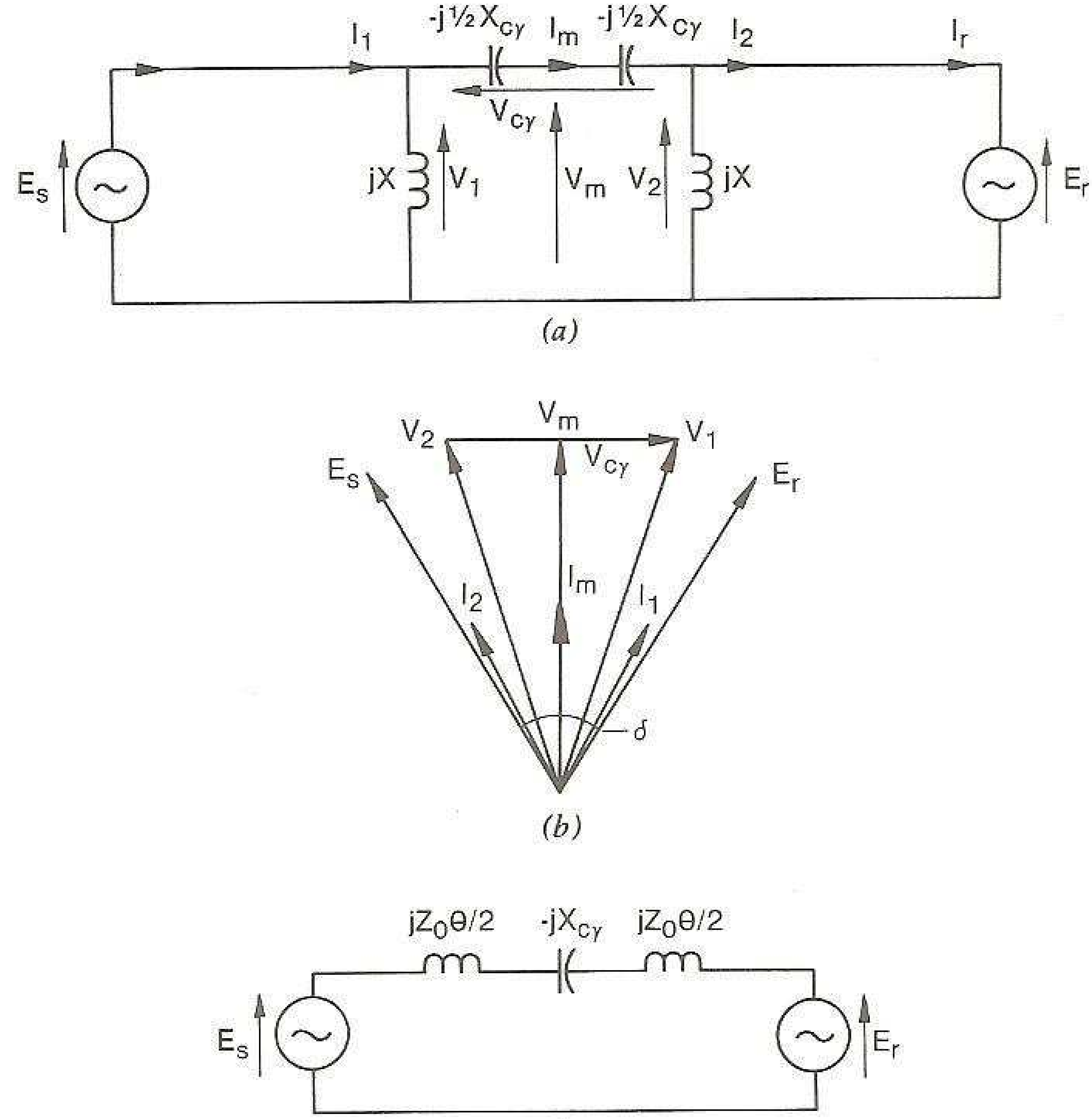


FIGURE 29. (*a*) Symmetrical line with midpoint series capacitor and shunt reactors. (*b*) General phasor diagram for Figure 29*a*. (*c*) Equivalent circuit with perfect shunt compensation.

Power-Transfer Characteristics and Maximum Transmissible Power. The general phasor diagram is shown in Figure 29*b*. Considering the left-hand (sending-end) line section, the conditions at its two ends are related by the fundamental Equation 2:

$$\mathbf{E}_s = \mathbf{V}_1 \cos\frac{\theta}{2} + jZ_0\mathbf{I}_1 \sin\frac{\theta}{2} \qquad (99a)$$

$$\mathbf{I}_s = j\frac{\mathbf{V}_1}{Z_0} \sin\frac{\theta}{2} + \mathbf{I}_1 \cos\frac{\theta}{2}. \qquad (99b)$$

The receiving-end half of the line behaves similarly. The capacitor reactance is $X_{c\gamma} = 1/\omega C_\gamma$ and the voltage across the capacitor is given by

$$\mathbf{V}_{c\gamma} = \mathbf{V}_1 - \mathbf{V}_2 = -j\mathbf{I}_m X_{c\gamma}. \qquad (100)$$

By symmetry, $P = V_m I_m$, $E_s = E_r$, and

$$\mathbf{V}_m = \mathbf{V}_1 - \tfrac{1}{2}\mathbf{V}_{c\gamma} = \mathbf{V}_2 + \tfrac{1}{2}\mathbf{V}_{c\gamma}. \qquad (101)$$

The currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are given by

$$\mathbf{I}_m = \mathbf{I}_1 + \frac{j\mathbf{V}_1}{X} = \mathbf{I}_2 - j\frac{\mathbf{V}_2}{X} . \tag{102}$$

Using these relationships, and taking $\mathbf{V}_m$ as reference phasor, it is possible to follow the same procedure as was used in Section 2.2.6 to derive the basic power-transfer characteristic (Equation 37). This yields the following results:

$$P = \frac{E_s V_m}{Z_0 \sin\frac{\theta}{2} - \frac{X_{c\gamma}}{2}\left[\cos\frac{\theta}{2} + \frac{Z_0}{X}\sin\frac{\theta}{2}\right]} \sin\frac{\delta}{2} \tag{103}$$

and

$$E_s \cos\frac{\delta}{2} = V_m\left[\cos\frac{\theta}{2} + \frac{Z_0}{X}\sin\frac{\theta}{2}\right] = E_r \cos\frac{\delta}{2} . \tag{104}$$

If V_m is substituted from Equation 104 into Equation 103, the following general result is obtained for the symmetrical line, if $E_s = E_r$:

$$P = \frac{E_s E_r}{\left[Z_0 \sin\theta - \frac{X_{c\gamma}}{2}(1 + \cos\theta)\,\mu_x\right]\mu_x} \sin\delta , \tag{105}$$

where

$$\mu_x = 1 + \frac{Z_0}{X}\,\frac{\sin\theta}{1 + \cos\theta} = 1 + \frac{Z_0}{X}\tan\frac{\theta}{2} . \tag{106}$$

In the absence of the shunt reactors, $\mu_x = 1$. With fixed terminal voltages, $E_s = E_r = E$, the transmission angle δ can be determined from Equation 105 for any level of power transmission below the maximum. Once δ is known, V_m can be determined from Equation 104. Then $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{V}_{c\gamma}$, and other quantities can be found from Equations 99 through 101.

Special Cases:

1. The most familiar form of the series-compensated power transfer characteristic is obtained from Equation 105 by ignoring the shunt capaci-

tance of the line and removing the shunt reactors. Then $Z_0 \sin\theta$ is replaced by X_l and $\mu_x = 1$, so that with $E_s = E_r = E$,

$$P = \frac{E^2}{X_l - X_{c\gamma}} \sin\delta . \quad (107)$$

If the degree of series compensation k_{se} is defined by

$$k_{se} = \frac{X_{c\gamma}}{X_l} = \frac{X_{c\gamma}}{\omega a l} , \quad (108)$$

then

$$P = \frac{E^2}{X_l(1 - k_{se})} \sin\delta , \quad (109)$$

which corresponds to Equation 72. This form of the power-transfer characteristic is useful because of its simplicity. The error resulting from the neglect of shunt capacitance is illustrated in the example of Section 2.5.3.

2. Another important special case arises when the shunt reactors are chosen to compensate the line capacitance perfectly as discussed in Section 2.4.1. Their reactances are then each

$$X = Z_0 \frac{\sin(\theta/2)}{1 - \cos(\theta/2)} . \quad (110)$$

From Equation 106 the shunt reactance factor μ_x reduces to

$$\mu_x = \sec\frac{\theta}{2} \quad (111)$$

and the power-transfer characteristic becomes (with $E_s = E_r = E$)

$$P = \frac{E^2}{2Z_0 \sin\dfrac{\theta}{2} - X_{c\gamma}} \sin\delta . \quad (112)$$

The perfect shunt compensation of the capacitance of each half of the line leaves only the series reactance in the equivalent circuit, which is shown in Figure 29*c*. This equivalent circuit does not take into account the fact that because the shunt-inductive compensation is concentrated, the line voltage profile is not perfectly flat, even at no-load.

The maximum transmissible power computed from Equation 105 can be compared with the value obtained with uniformly distributed shunt and series compensation (Equation 70). Reference 2 describes this comparison for the scheme of Figure 29*a*, as well as for several other capacitor locations. (See also Section 2.5.3).

Reactive Power Requirements at the Terminals. The reactive power at the sending-end terminal is given by

$$Q_s = \mathrm{Im}(\mathbf{E}_s \mathbf{I}_s^*) . \tag{113}$$

From Equations 99 through 102, it can be shown that

$$\mathbf{E}_s = \mu_x V_m \cos\frac{\theta}{2} + j\frac{P}{V_m}\left[Z_0 \sin\frac{\theta}{2} - \frac{\mu_x X_{c\gamma}}{2}\cos\frac{\theta}{2}\right] \tag{114}$$

and

$$\mathbf{I}_s = I_m\left[\frac{\mu'_x X_{c\gamma}}{2Z_0}\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right] + j\frac{\mu_x V_m}{Z_0}\sin\frac{\theta}{2}, \tag{115}$$

where

$$\mu'_x = 1 - \frac{Z_0}{X}\frac{1+\cos\theta}{\sin\theta} = 1 - \frac{Z_0}{X}\operatorname{ctg}\frac{\theta}{2} . \tag{116}$$

In the absence of the shunt reactors, $\mu'_x = 1\ (= \mu_x)$. Substituting for $\mathbf{E}_s$ and $\mathbf{I}_s^*$ in Equation 113, the result is

$$Q_s = \frac{P_0}{2}\left(\frac{P}{P_0}\right)^2\left(\frac{V_0}{V_m}\right)^2\left\{\frac{X_{c\gamma}}{2Z_0}\left[(\mu'_x - \mu_x) - (\mu'_x + \mu_x)\cos\theta\right]\right.$$
$$\left. + \left[1 - \left(\frac{X_{c\gamma}}{2Z_0}\right)^2 \mu'_x\mu_x\right]\sin\theta\right\} - \frac{P_0}{2}\left(\frac{V_m}{V_0}\right)^2 \mu'_x\mu_x \sin\theta . \tag{117}$$

By symmetry the reactive power at the receiving end is $Q_r = -Q_s$. In all cases V_m is given by Equation 104, with δ derived from Equation 105.

Special Cases:

1. In the absence of the shunt reactors $\mu'_x = \mu_x = 1$ and the Equation 117 for Q_s and $-Q_r$ reduces to

$$Q_s = \frac{P_0}{2}\left\{\sin\theta\left\{\left(\frac{P}{P_0}\right)^2\left(\frac{V_0}{V_m}\right)^2\left[1-\left(\frac{X_{c\gamma}}{2Z_0}\right)^2\right]-\left(\frac{V_m}{V_0}\right)^2\right\}\right.$$

$$\left.-\cos\theta\left(\frac{P}{P_0}\right)^2\left(\frac{V_0}{V_m}\right)^2\frac{X_{c\gamma}}{Z_0}\right\}. \tag{118}$$

In the absence of the series capacitor and the shunt reactors, this equation reduces to Equation 29 for the uncompensated line.

2. If the shunt reactors are designed for perfect no-load compensation (Equation 110), then μ_x is given by Equation 111 and μ'_x by

$$\mu'_x = \frac{Z_0}{X}\csc\frac{\theta}{2}. \tag{119}$$

Then the following simplifications arise:

$$(\mu'_x - \mu_x) - (\mu'_x + \mu_x)\cos\theta = 2\left[1 - 2\cos\frac{\theta}{2}\right] \tag{120}$$

and

$$\mu'_x\mu_x = \frac{2Z_0}{X\sin\theta}. \tag{121}$$

The terminal reactive powers become

$$Q_s = \frac{P_0}{2}\left(\frac{P}{P_0}\right)^2\left(\frac{V_0}{V_m}\right)^2\left\{\frac{X_{c\gamma}}{Z_0}\left(1 - 2\cos\frac{\theta}{2}\right)\right.$$

$$\left.+\left[1-\left(\frac{X_{c\gamma}}{2Z_0}\right)^2\frac{2Z_0}{X\sin\theta}\right]\sin\theta\right\} - \frac{P_0}{2}\left(\frac{V_m}{V_0}\right)^2\frac{2Z_0}{X} \tag{122}$$

with $Q_r = -Q_s$. Note that at no-load ($P = 0$), Q_s reduces to $-V_m^2/X$; that is, the reactive power absorption at the sending end is identical to that in the left-hand shunt reactor. (See Section 2.4.1). Similarly, at no-load the receiving-end absorption equals the reactive power in the right-hand shunt reactor, both being equal to V_m^2/X. At no-load there is no current in the series capacitor and the two halves of the line behave as two back-to-back open-circuited lines each with a shunt reactor at its receiving end.

2.5.3. Example of a Series-Compensated Line

The following example shows the mechanism of series compensation in the steady state. In order to demonstrate the importance of the accompanying shunt reactors, the example is worked with and without them.

It is again emphasized that the midpoint is only one of many possible locations for the series capacitor, and is not necessarily always the most favorable, either technically or economically. For a comparative steady-state evaluation of different capacitor locations, the reader is referred to Reference 2.

Midpoint Series Capacitor Without Shunt Reactors. The example is for a 400-mi line, in order to facilitate comparisons with examples elsewhere in the chapter.† With $a =$ 400 mi, the electrical length is $\theta = \beta a = 0.8108$ rad or 46.5°. In the per-unit system based on V_0 and Z_0, the per-unit reactance is $X_l = 0.8108$ and the total shunt capacitive susceptance is $B_c = 0.8108$ pu. The series compensating capacitor is chosen so as to compensate 50% of the line reactance, so that $X_{c\gamma} = 0.5 \times 0.8108 = 0.4054$ pu. (At 500 kV with $Z_0 = 250\ \Omega$, $X_{c\gamma} = 101.4\ \Omega$.) Without midpoint shunt reactors, $X = \infty$ and $\mu_x = \mu'_x = 1$. Assuming that the terminal voltages are constant, $E_s = E_r = V_0 = 1.0$ pu. From Equation 104 the no-load midpoint voltage (with $\delta = 0$) is $1/\cos(\theta/2) = 1.0882$ pu. Since there is no current through the series capacitor, this voltage appears also on both sides of it. From Equation 105 the power transfer characteristic is given by

$$P = \frac{\sin\delta}{\sin 0.8108 - \tfrac{1}{2} \times 0.4054(1 + \cos 0.8108)}$$

$$= 2.6144 \sin\delta . \tag{123}$$

If there were no compensation P_{max} would be 1.3796 pu (from Equation 37), so the series capacitor increases P_{max} by a factor of 2.6144/1.3796 = 1.8950. If the compensation were uniformly distributed with $k_{se} = 0.5$, P_{max} would be 2.6072 P_0 (from Equation 70), which is only about 0.25% different from the value obtained here with lumped capacitors.

† Note that the full distributed-parameter representation has been used for both halves of the line, and not the equivalent-π circuits as in Sections 2.4 and 2.6.

The reactive power requirements at the terminals can be calculated from Equation 118, which gives

$$Q_s = -Q_r = \left[0.2079\frac{p^2}{v_m^2} - 0.3624\, v_m^2\right] P_0\,, \tag{124}$$

where $p = P/P_0$ and $v_m = V_m/V_0$. From Equations 100 and 101 the voltage on either side of the capacitor ($V_1 = V_2$) is given by

$$V_1 = \left| V_m - j\,0.2027\,\frac{P}{V_m} \right| . \tag{125}$$

The voltage across the series capacitor is given by

$$\mathbf{V}_{c\gamma} = -jI_m X_{c\gamma} = -j\,0.4054\,\frac{P}{V_m}. \tag{126}$$

Table 5 shows the variation of these parameters as the power transfer increases from zero through 2.0 pu. Also shown is the variation of the transmission angle that would be obtained without compensation. For a given power transfer, this is about twice the value obtained with the series capacitor installed.

Although there is a marked reduction in transmission angle, the voltage V_1 ($= V_2$) on either side of the series capacitor is rather high, and there is very little reduction as the power increases. In addition there is appreciable absorption of reactive power at the line ends, even at the natural load P_0. This can be associated with the generally high voltage conditions along the line, together with the fact that the series capacitor itself generates reactive power.

Midpoint Series Capacitor with Shunt Reactors. The high voltage and reactive power absorption at the ends of the line can be corrected by means of shunt reactors. The value of X required on either side of the series capacitor is given by Equation 110 as $X = 4.8656\ Z_0$. From Equations 119 and 111, $\mu_x' = 0.5211$ and $\mu_x = 1.0882$. Substituting in Equation 105, or directly from Equation 112,

$$P = 2.6084 \sin \delta\,. \tag{127}$$

The maximum transmissible power is hardly affected by the addition of the shunt reactors. With uniformly distributed shunt compensation $k_{sh} = 1$ and from Equation 72, $P = P_0/0.8108(1-0.5) = 2.4667\ P_0$. The midpoint voltage V_m is given by Equation 104 as

$$V_m = 1.0 \cos\frac{\delta}{2}\,, \tag{128}$$

TABLE 5
Performance of Midpoint Series Compensated Line without Shunt Reactors
(a = 400 mi, k_{sc} = 0.5)

$p = \frac{P}{P_0}$	$v_m = \frac{V_m}{V_0}$	Terminal Reactive Power $\frac{Q_s}{P_0} = -\frac{Q_r}{P_0}$	Trans-Mission Angle δ (°)	$\frac{V_1}{V_0}$	$\frac{V_{c\gamma}}{V_0}$	δ Without Compen-sation (°)
0	1.0882	−0.4292	0	1.0882	0	0
0.25	1.0870	−0.4172	5.487	1.0880	0.0932	10.440
0.50	1.0832	−0.3809	11.026	1.0872	0.1871	21.249
0.75	1.0767	−0.3193	16.671	1.0859	0.2824	32.932
1.00	1.0673	−0.2303	22.488	1.0841	0.3798	46.456
1.25	1.0546	−0.1109	28.563	1.0816	0.4805	64.966
1.50	1.0378	0.0440	35.011	1.0784	0.5860	unstable
2.00	0.9866	0.5015	49.906	1.0688	0.8218	unstable

which indicates a flat voltage profile at no-load (when $\delta = 0$). The reactive power requirements are given by Equation 122 or 117 as

$$Q_s = -Q_r = P_0\left[0.1841\frac{p^2}{v_m^2} - 0.2055\, v_m^2\right]. \tag{129}$$

The voltage on either side of the series capacitor ($V_1 = V_2$) is the same as the voltage across the shunt reactors and is given again by

$$V_1 = \left| V_m - j\,0.2027\,\frac{P}{V_m}\right|, \tag{130}$$

and the capacitor voltage by

$$\mathbf{V}_{c\gamma} = -j\,0.4054\,\frac{P}{V_m}. \tag{131}$$

Table 6 shows the variation of these parameters as the power transfer increases from zero through 2 P_0.

The voltage on either side of the series capacitor is nearly 1.0 pu over practically the whole range of power transfer, implying that the shunt reactors could be permanently connected without disadvantage. The reactive absorption at the terminals is considerably reduced, and at high values of P the terminal power factors become lagging, which may be beneficial to transient stability because the internal generator voltages are increased.

Note that shunt reactors have not been connected at the line ends, although it is quite practical to do this. In the calculations the generators at the ends fulfill the shunt compensating function, and at no-load each absorbs exactly the same amount of reactive power as one of the central shunt reactors,—that is, the line-charging reactive power of one 100-mi section (=0.2055 P_0).

TABLE 6
Performance of Midpoint Series Compensated Line with Shunt Reactors
(a = 400 mi, k_{se} = 0.5, Shunt Reactor X = 4.8656 Z_0)

$p = \frac{P}{P_0}$	$v_m = \frac{V_m}{V_0}$	Terminal Reactive Power $\frac{Q_s}{P_0} = -\frac{Q_r}{P_0}$	Transmission Angle δ (°)	$\frac{V_1}{V_0}$	$\frac{V_{cy}}{V_0}$	δ Without Compensation (°)
0	1	−0.2055	0	1	0	0
0.25	0.9988	−0.1935	5.500	1.000	0.1015	10.440
0.50	0.9954	−0.1572	11.051	1.001	0.2036	21.249
0.75	0.9894	−0.0954	16.710	1.001	0.3073	32.932
1.00	0.9807	−0.0062	22.543	1.002	0.4134	46.456
1.25	0.9689	0.1135	28.635	1.004	0.5230	64.966
1.50	0.9534	0.2689	35.104	1.005	0.6378	unstable
2.00	0.9061	0.7282	50.063	1.011	0.8948	unstable

2.6. COMPENSATION BY SECTIONING[15-20] (DYNAMIC SHUNT COMPENSATION)

2.6.1. Fundamental Concepts

If a synchronous machine is connected at an intermediate point along a transmission line, it can maintain constant the voltage at that point, just as the terminal synchronous machines do at the ends. In so doing, it divides the line into two sections which are apparently quite independent. The voltage profile, the maximum transmissible power, and the reactive power requirements of each section can then be determined separately—the problems in each independent section being less severe than in the line as a whole. The maximum transmissible power in particular is now determined by the "weakest link in the chain," which is generally the longest section. For example, if a line is sectioned into two equal halves, then if shunt capacitance is neglected (or completely compensated by shunt reactors), the power transmission characteristic is given by Equation 38 for each half of the line separately. Thus, replacing δ by $\delta/2$ and X_l by $X_l/2$, with $E_s = E_r = E$,

$$P = 2\frac{E_m E}{X_l}\sin\frac{\delta}{2}, \tag{132}$$

where E_m is the midpoint voltage, held constant by the synchronous machine or by some other constant-voltage compensating device. The maximum transmissible power is doubled.

This scheme, called "compensation by sectioning," was proposed by F. G. Baum in 1921 (See Reference 3). Baum suggested that by connecting synchronous condensers at intervals of about 100 mi, a substantially flat voltage profile could be achieved at all levels of power transmission. The condensers would adjust the virtual natural load, P_0', to be equal to the actual load at all times. Baum calculated an example of an 800-mi line at 220 kV transmitting about 100 MW (approximately 0.8 P_0) *with a total transmission angle of about* 146°. Although Baum considered voltage regulation and reactive power requirements in detail, he did not consider stability. It was not until much later that the stability of long lines compensated in this way was adequately understood for practical use (see Section 2.3.4).

If losses are neglected the "compensating" current taken by the intermediate synchronous machine is purely reactive (i.e., in phase quadrature with the voltage at the point of connection), and the machine supplies or absorbs only reactive power to or from the line. In the steady state, therefore, the machine can maintain constant voltage at its point of connection *without requiring a mechanical prime mover.* In a given steady state there is a certain ratio between the compensating current $\mathbf{I}_\gamma$ and the voltage $\mathbf{V}$ at the point of connection. The ratio has the dimensions of a susceptance, which is capacitive if $\mathbf{I}_\gamma$ leads $\mathbf{V}$ and inductive if $\mathbf{V}$ leads $\mathbf{I}_\gamma$. This implies that the synchronous machine could, in the steady state, be replaced by a capacitor or a reactor.

However, if the power being transmitted along the line changed value, the voltage $\mathbf{V}$ would tend to change. In order to restore $\mathbf{V}$ to the constant value required to maintain the independence of the line sections, the capacitive or inductive susceptance would have to change also. This suggests that if the susceptance of a real capacitor or reactor could be modulated or controlled in such a way as to maintain constant voltage at its point of connection, the device would be functionally equivalent to the synchronous machine. Figure 30 illustrates the principle of modulating the susceptance in such a way as to maintain constant terminal voltage.

So far it has been implied that the shunt compensating device must maintain constant voltage *magnitude* at its point of connection. Under steady-state or very slowly varying conditions, the *static* compensator (i.e., a compensator having no moving parts) can be made to be functionally equivalent to an intermediate synchronous machine. Under more rapidly varying conditions, the inertia of the synchronous machine rotor influences the *phase* of the voltage at the point of connection, because of the exchange of kinetic energy between it and the system as the rotor accelerates or decelerates. The purely static compensator cannot exchange this energy with the system, and this leads to a different influence on the system under rapidly varying conditions.†

This section develops the theory of compensation by sectioning in the steady state and for very slowly varying conditions, that is, slowly enough for the rates of change of kinetic energy of rotating machines to be negligible. In spite of this, it is seen that in certain regimes compensation by sectioning is an essentially *dynamic* process (in the control engineer's sense).

2.6.2. Dynamic Working of the Midpoint Compensator

The *fixed* midpoint shunt susceptance was considered in Section 2.4.3. In this section the same representation of the transmission line is used, a π-

† It is true that the static compensator contains capacitors and inductors which can store energy, but because they carry alternating current there is no significant net storage of energy averaged over a period long enough for a significant change to take place in the kinetic energy of the synchronous machines in the system (typically a few cycles).

An exception to this is the static compensator constructed of a very large dc energy-storage coil connected to the line via a three-phase bridge rectifier.[21] To obtain a useful amount of stored energy, with acceptable losses, the storage coil would probably need to be superconducting.

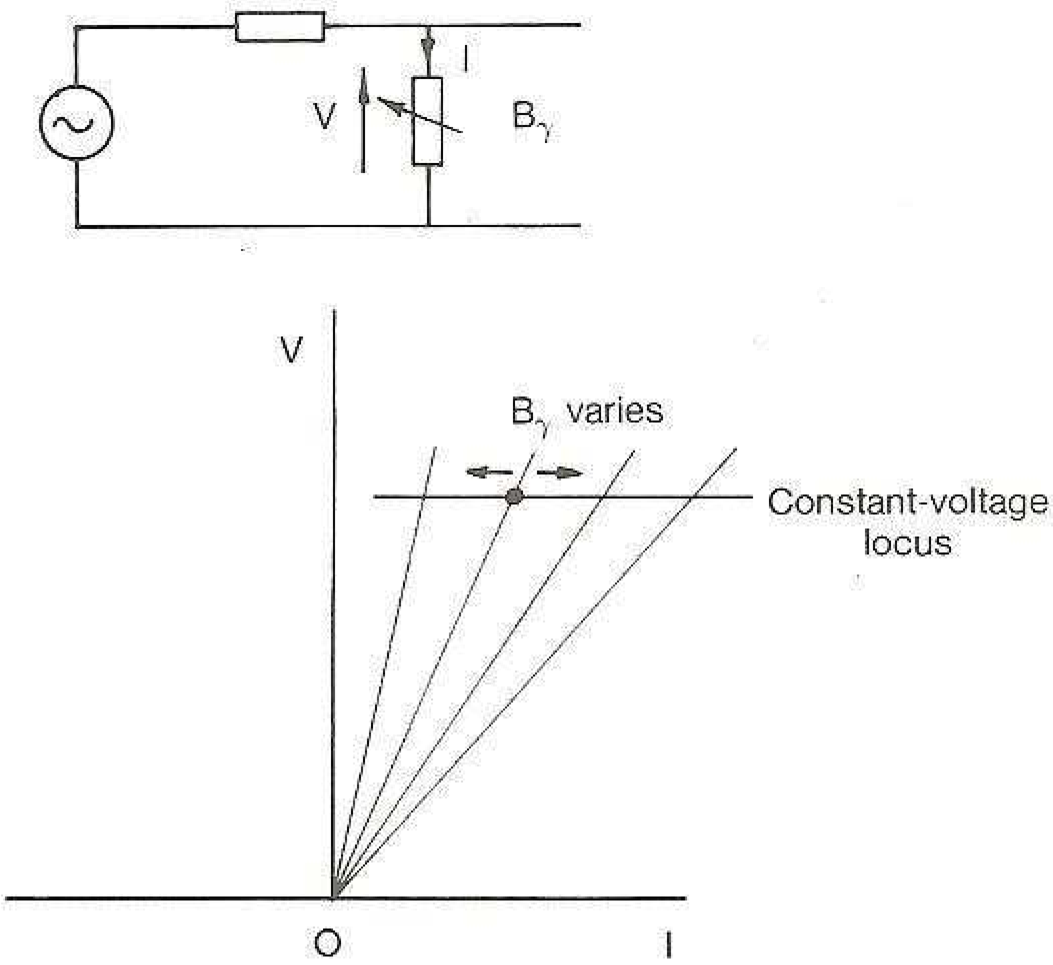


FIGURE 30. Principle of maintaining constant ac voltage at the terminals of a controlled susceptance.

equivalent circuit for each half, as in Figure 27*a*. It is assumed here, however, that the compensating susceptance is continuously controlled in such a way as to keep the midpoint voltage constant with a value E_m. The analysis applies equally well with a synchronous condenser instead of a controlled susceptance.

Now the midpoint voltage is related to the terminal voltages in Equation 92, so it follows that with $E_s = E_r = E$,

$$1 - s = \frac{E}{E_m} \cos \frac{\delta}{2}. \tag{133}$$

At this stage it is not of interest to interpret s as an equivalent degree of series compensation. Instead it is used to obtain the compensating susceptance B_γ necessary to satisfy Equation 133. Thus, (substituting from Equations 86 and 91).

$$B_\gamma = -\frac{4}{X_l}\left[1 - \frac{E}{E_m} \cos \frac{\delta}{2}\right] + \frac{B_c}{2}, \tag{134}$$

where B_c is the total shunt capacitive susceptance of the whole line and X_l is its total reactance, as in Figure 27*a*. This equation tells how B_γ must vary with the transmission angle δ in order to maintain the midpoint voltage equal to E_m. Naturally, through δ, B_γ varies with the power being transmitted.

Since s now *varies* with δ (Equation 133), the power transmission characteristic is modified. From Equation 90,

$$P = \frac{E^2}{X_l(1-s)} \sin\delta = \frac{E_m E}{X_l \cos(\delta/2)} \sin\delta, \tag{135a}$$

that is,

$$P = 2\frac{E_m E}{X_l} \sin\frac{\delta}{2}. \tag{135b}$$

This is identical to Equation 132. It implies that in the steady state the line is sectioned into two independent halves.

If $E_m = E$, the power transmission characteristic is given by

$$P = \frac{2E^2}{X_l} \sin\frac{\delta}{2}. \tag{136}$$

This is shown as the upper curve in Figure 31. The maximum transmissible power is $2E^2/X_l$, twice the steady-state limit of the uncompensated line. It is reached when $\delta/2 = \pi/2$, that is, with a transmission angle of 90° across each half of the line, and a total transmission angle δ of 180° across the whole line.

(a) Illustration of Dynamic Working. The power transmission characteristic expressed by Equation 90 can be interpreted as a sinusoid whose amplitude P'_{max} varies as s varies. From Equation 135a, P'_{max} varies also

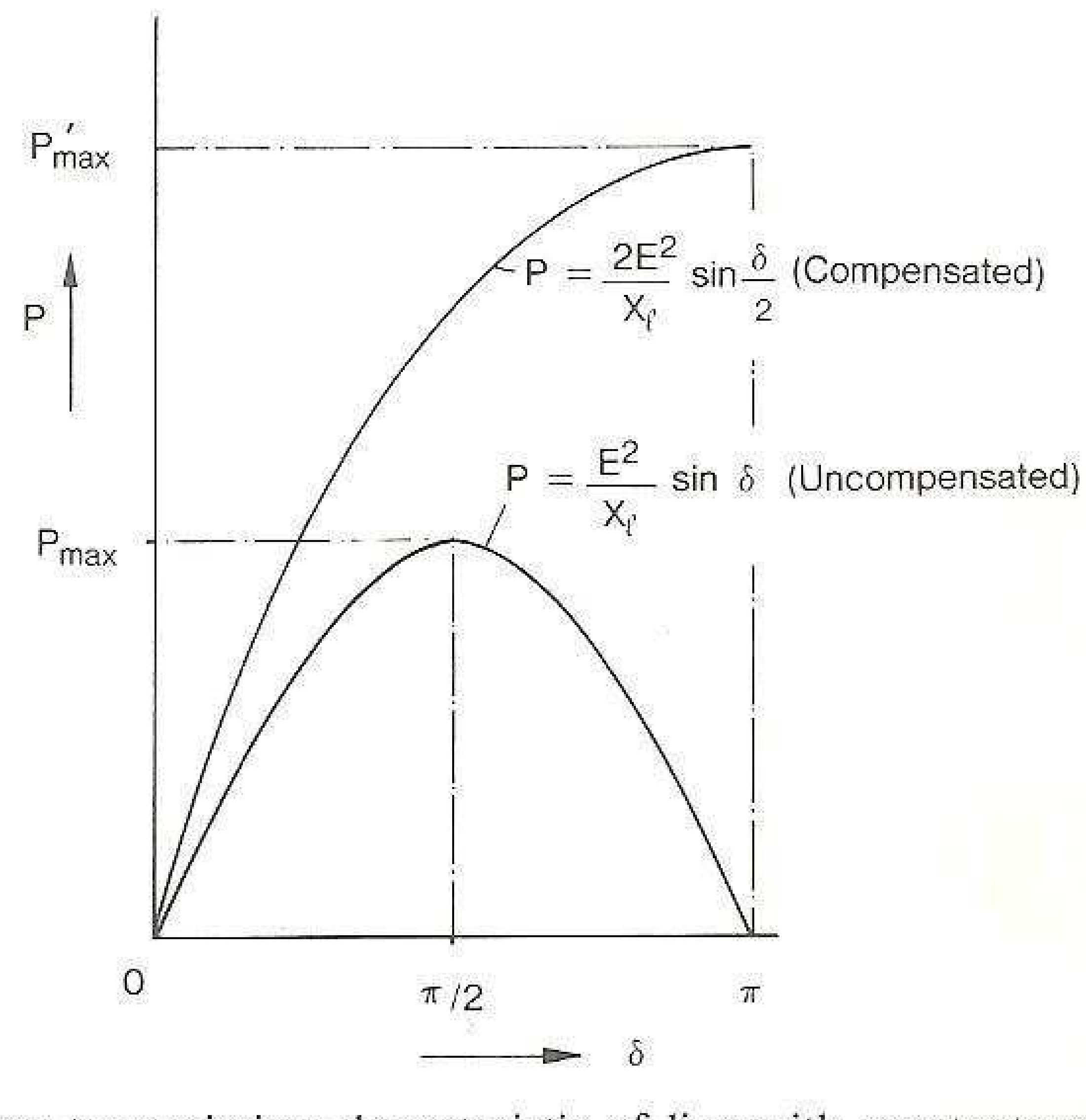


FIGURE 31. Power transmission characteristic of line with constant-voltage compensator at midpoint. ($E_m = E$)

with δ itself and, therefore, also with P (through Equation 135b). This can be written

$$P = P'_{\max} \sin \delta \,, \tag{137}$$

where

$$P'_{\max} = \frac{E_m E}{X_l \cos (\delta/2)} \,. \tag{138}$$

(δ can always be determined from Equation 135b, because E, E_m and X_l are constants.) This gives rise to the concept of a series of sinusoids with varying amplitudes $P'_{\max}$ as shown in Figure 32 (dotted curves). Each corresponds to a fixed value of the compensating susceptance B_γ. For example, suppose P is equal to the steady-state limit of the uncompensated line, $P_{\max} = E^2/X_l$. From Equation 136 with $E_m = E$, $\delta = 2 \sin^{-1} (½) = 60°$. Operation is at the point A in Figure 32. From Equation 138, $P'_{\max} = P_{\max}/\cos(30°) = 1.155\ P_{\max}$; that is, the "current" static power transmission characteristic has a maximum transmissible power of 1.155 $P_{\max}$.

If the transmitted power were increased to 1.2 $P_{\max}$, δ would increase to $2 \sin^{-1} (1.2/2) = 73.74°$, and $P'_{\max}$ would increase to $P_{\max}/\cos (36.87°) = 1.250\ P_{\max}$. Operation is at the point B in Figure 32. The

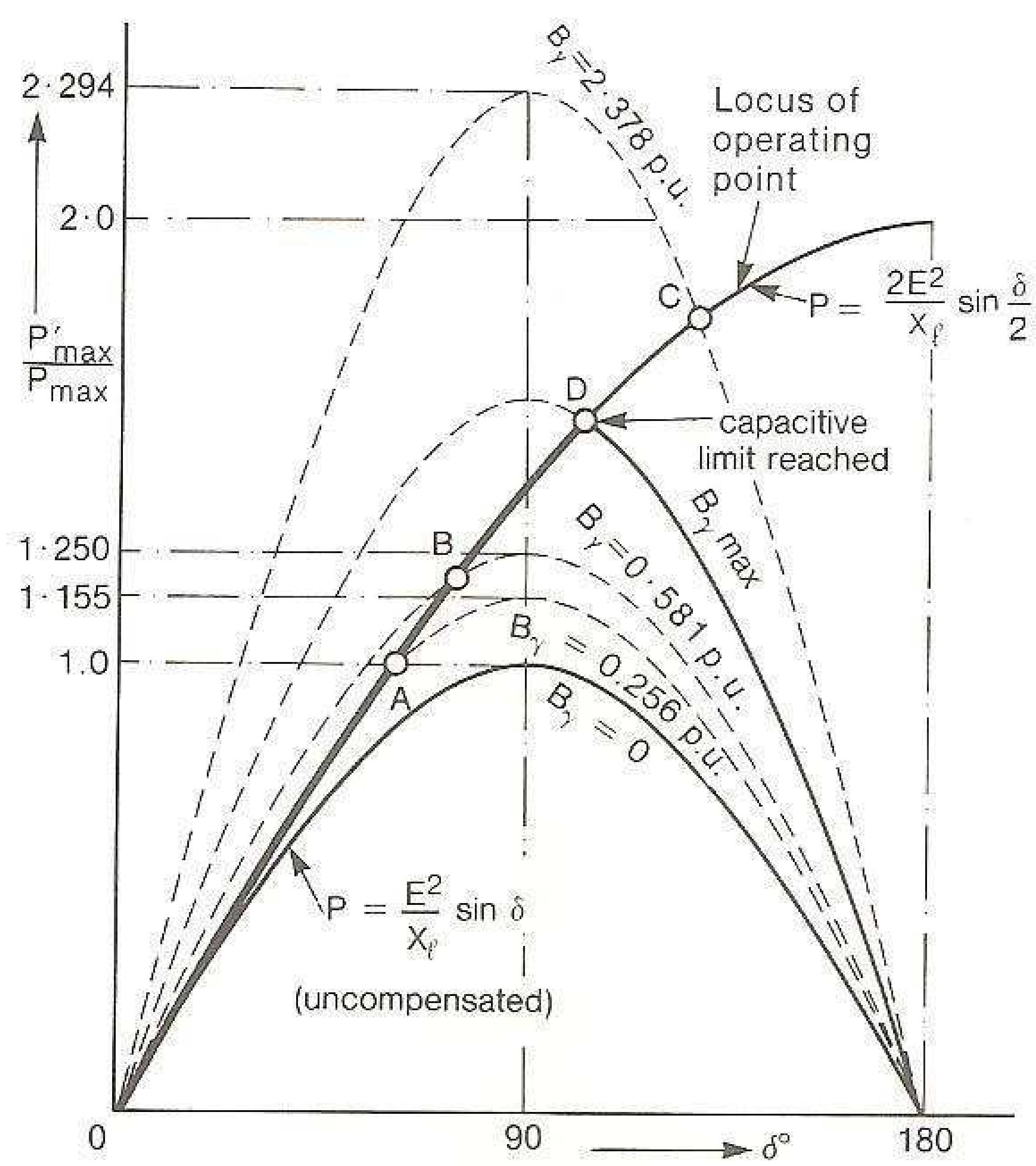


FIGURE 32. Dynamic working of midpoint shunt compensator.

controlling effect of the midpoint compensator constrains the operating point to move along the locus given by Equation 136, passing smoothly from one constant-B_γ sinusoid to another as P varies.

For $P > \sqrt{2}\, P_{max}$, δ is greater than 90°. The system is stable as long as $dP/d\delta > 0$ (i.e., as long as an increase in transmission angle is accompanied by an increase in transmitted power; see Section 2.2.6). However, the operating point is now on the unstable side of the "current" constant-B_γ sinewave. For example, if $P = 1.8\, P_{max}$, $\delta = 2 \sin^{-1} (1.8/2) = 128.32°$, and $P'_{max} = 2.294\, P_{max}$. Operation is at point C in Figure 32. The system owes its stability to the fact that if the transmission angle δ increases slightly, the compensator responds by changing B_γ immediately in such a way as to keep V_m constant and so to increase P'_{max} to the value which satisfies Equation 138, so that the operating point moves along the stable characteristic, $P = 2P_{max} \sin (\delta/2)$.

Typically, in transmission systems which are compensated by sectioning and which are operating at high power levels, the effective compensating susceptance B_γ is capacitive. In practice there is an economic limit to the capacitive reactive power of the compensator. With many types of compensator, when the limit is reached the compensator ceases to maintain constant voltage at its terminals and behaves instead like a fixed susceptance. The operating point then leaves the dynamically stable characteristic $P = 2\, P_{max} \sin (\delta/2)$, and moves on to the constant-B_γ sinusoid corresponding to the maximum value of B_γ, that is, $B_{\gamma max}$.† This departure is shown at point D in Figure 32. The higher the value of $B_{\gamma max}$, the steeper is the (negative) slope of the constant-B_γ characteristic at the point of departure. If $B_{\gamma max}$ were smaller, the point of departure could occur on the stable side of one of the dotted sinusoids in Figure 32, such as at point A or B. However, the power transmitted P and the transmission angle δ would then have to be kept to much lower values. In other words, the capacitive current rating of the compensator is one of the main factors limiting the achievable increase in the maximum transmissible power.

(b) Reactive Power Requirements at the Terminals. From Equations 95, 133, and 134 it can be shown that the reactive power requirement at the sending end is given by

$$Q_s = \frac{2E^2}{X_l} \left[\left(1 - \frac{X_l B_c}{8} \right) - \frac{E_m}{E} \cos \frac{\delta}{2} \right] \tag{139}$$

with $E_s = E_r = E$. By symmetry, $Q_r = -Q_s$.

† For a synchronous condenser or saturated-reactor compensator $B_{\gamma max}$ is the maximum capacitive current divided by E_m.

(c) Compensator Control and Reactive Power Requirements. A practical control system for varying B_γ would not be based on Equation 134, because there is no information as to the value of δ available at the intermediate compensator station. Instead, a feedback control system would be used to keep $V_m = E_m$. In the saturated-reactor compensator an inherent regulating process achieves the same end.

With constant $V_m = E_m$, the compensator reactive power $Q_\gamma = V_m^2 B_\gamma$ is a function of δ also:

$$Q_\gamma = V_m^2 B_\gamma = -E_m^2\left\{\frac{4}{X_l}\left[1 - \frac{E}{E_m}\cos\frac{\delta}{2}\right] - \frac{B_c}{2}\right\} \tag{140}$$

Through the parameter δ, Q_γ is related to the transmitted power P by Equations 135 and 140. Note that $X_l B_c = \omega^2 a^2 lc = \theta^2$, θ being the electrical length of the line in radians.

2.6.3. Example of Line Compensated by Sectioning†

As an example of compensation by sectioning, or "dynamic shunt compensation," consider a 400-mi line with a midpoint constant-voltage compensator. As in Section 2.5.3, $X_l = 0.8108\ Z_0$ and $B_c = 0.8108/Z_0$. The power transmission characteristic is given by Equation 132 as

$$P = \frac{2\times 1^2}{0.8108}\sin\frac{\delta}{2}P_0 = 2.4667\ P_0 \sin\frac{\delta}{2} \tag{141}$$

with $E_s = E_r = E = 1.0$ pu. P'_{max} is quite close to the value 2.6084 P_0 achieved with 50% series compensation (with shunt reactors connected). The reactive power requirement of the compensator is given by Equation 140 as

$$Q_\gamma = -\left[4.9334\left(1 - \cos\frac{\delta}{2}\right) - 0.4054\right] P_0 \tag{142}$$

and the terminal reactive powers are given by Equation 139 as

$$Q_s = -Q_r = 2.4667\left(0.9178 - \cos\frac{\delta}{2}\right) P_0\,. \tag{143}$$

† The two halves of the line are again represented by equivalent-π circuits.

At no-load $Q_s = -0.2027\ P_0$, corresponding to the line-charging reactive power of the 100 mi of line nearest to the sending end. (Note that this differs from the value $-0.2055\ P_0$ calculated from Equation 30. This is due to the approximation involved in the π-equivalent circuit.) At no-load, $Q_\gamma = 0.4054\ P_0$, corresponding to the line-charging reactive power of the central 200 mi of the line.

The variation of the main parameters is shown in Table 7 as the transmitted power varies. It can be seen that the reactive power absorbed or supplied by each terminal is just half that of the compensator. The two vary in concert, the terminal synchronous machines compensating the extreme 100-mi portions of the line, while the compensator compensates the central 200-mi portion.

Although the overall transmission angle δ exceeds 90°, transmission remains stable up to 180°, as long as the midpoint voltage remains constant. It can be seen that the power transmission up to 1.25 P_0 does not require excessive capacitive reactive power from the compensator.

TABLE 7
Example of a 400-Mile Transmission Line Compensated by a Constant-Voltage Device at its Midpoint[a]

$p = \frac{P}{P_0}$	E_m	$\frac{Q_s}{P_0} = -\frac{Q_r}{P_0}$	$-Q_\gamma/P_0$	δ (°)	δ Without Compensation (°)
0	1.000	−0.2027	−0.4054	0	0
0.25	1.000	−0.1900	−0.3800	11.634	10.440
0.50	1.000	−0.1515	−0.3030	23.390	21.249
0.75	1.000	−0.0859	−0.1718	35.402	32.932
1.00	1.000	0.0091	0.0182	47.832	46.456
1.25	1.000	0.1375	0.2749	60.895	64.966
1.50	1.000	0.3058	0.6116	74.905	unstable
1.75	1.000	0.5256	1.0512	90.380	unstable
2.00	1.000	0.8202	1.6404	108.34	unstable
2.25	1.000	1.2530	2.5060	131.60	unstable
2.4667	1.000	2.2640	4.5280	180.00	unstable

[a] Note the opposite sign conventions for Q_s and Q_γ: Q_s and $-Q_\gamma$ are both negative for absorption; the compensator is then inductive. Q_s and $-Q_\gamma$ are both positive for generation; the compensator is then capacitive.

Whether the system has transient stability for major faults at this level of transmission is another question, which will be dealt with in the next chapter.

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